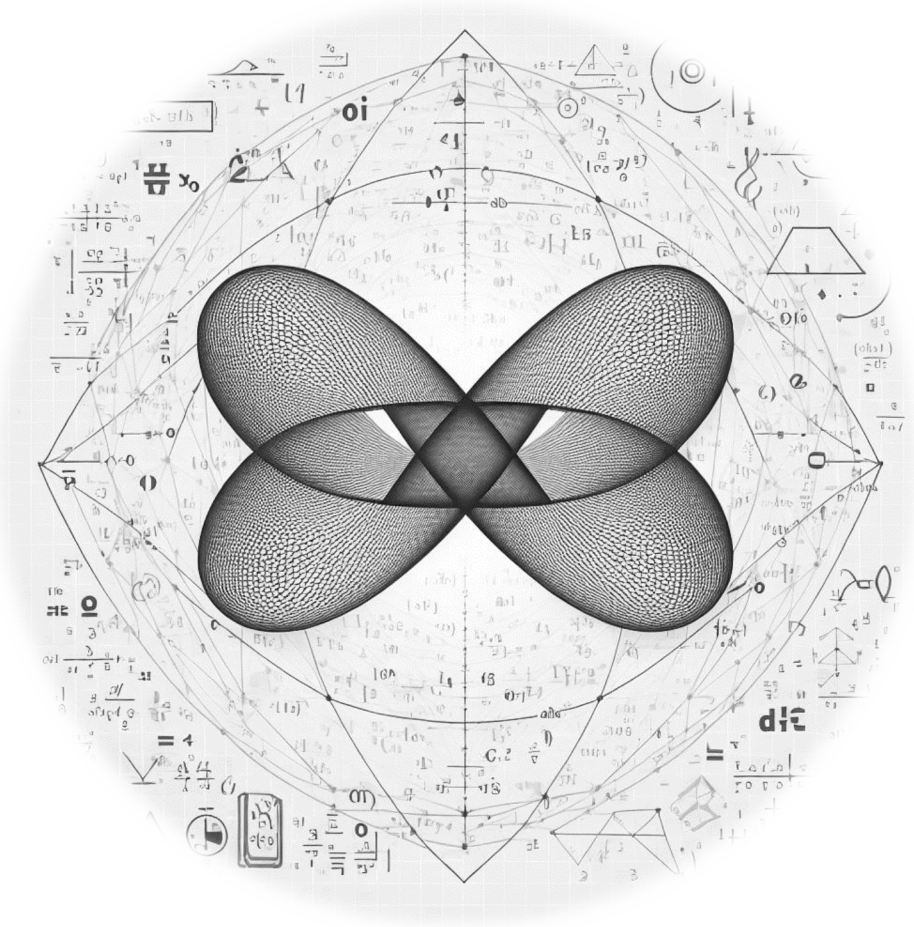
Contents

Unit 1. Linear Equations Foundations (Level 1)



1-1. Linear equations in 1 variable

Definition:

A linear equation in one variable is an equation of the form $ax + b = c$, where a, b, and c are constants, and $a \neq 0$.

Theory:

The goal of solving a linear equation is to find the value of the variable x that makes the equation true. This is done by isolating x on one side of the equation through a series of algebraic manipulations.

General Solution:

To solve ax + b = c, the formula to isolate x is: $x = \frac{c-b}{a}$

Steps to Solve a Linear Equation:

1. Isolate the Variable:

   Start by eliminating any constants added or subtracted to the variable term. This usually involves adding or subtracting the same value on both sides of the equation.

2. Solve for the Variable:

   Once the variable term is isolated, solve for the variable by performing the necessary arithmetic operation (e.g., division or multiplication).

$3y+7=22$ $5y-15=103y=15$	1) Solve for y:
2) Which equation has the same solution as the given equation? A) B) C) $3y=5$ D) $3y=7$	3) Jay walks at a speed of 4 miles per hour and runs at a speed of 6 miles per hour. He walks for w hours and runs for r hours for a combined total of 20 miles. Which equation represents this situation? A) $4w+6r=20$ B) $4w+6r=26$ C) $4w+6r=24$ D) $4w+6r=30$
4) Solve for x: $2x+3=9$ A) $x=3$ B) $x=6$ C) $x=2$ D) $x=0$	5) A teacher is creating an assignment worth 50 points. The assignment will consist of questions worth 2 points and questions worth 4 points. Which equation represents this situation, where x represents the number of 2-point questions and y represents the number of 4-point questions? A) $2x+4y=50$ B) $2(x+y)=50$ C) $2x+y=50$ D) $x+4y=50$

6) .Solve for t: $4t+8=2t+12$	7) Which expression is equivalent to $15m-(3m+2m)$? A) 5m B) 10m C) 12m D) 8m
8) If $7+p=13$, what is the value of $21+p$? *A*) 27 *B*) 30 *C*) 31 *D*) 32	9) Solve for x: $\frac{x^2}{16}=4$ *A*) 2 *B*) 8 *C*) 16 *D*) 32
10) 3 less than 9 times a number y is equal to 60. Which equation represents this situation? A) $9y-3=60$ B) $9y+3=60$ C) $3y-9=60$ D) $3-9y=60$	

Solutions:

1) Solve for x in the equation: $3x+5=11$	2) Solve for y: $5y-15=10$
1. Subtract 5 from both sides of the equation: $3x+5-5=11-5$ $3x=6$ 2. Divide both sides by 3: $x=\frac{6}{3} \Rightarrow x=2$	- $5y-15=10$ - $5y=25$ - $y=5$

3) Which equation has the same solution as the given equation? $3y+7=22$ A) $3y=15$ B) $3y=21$ C) $3y=5$ D) $3y=7$	4) Jay walks at a speed of 4 miles per hour and runs at a speed of 6 miles per hour. He walks for w hours and runs for r hours for a combined total of 20 miles. Which equation represents this situation? A) $4w+6r=20$ B) $4w+6r=26$ C) $4w+6r=24$ D) $4w+6r=30$
- $3y+7=22$ - $3y=15$ - Thus, the correct answer is A.	- $Distance=rate\times time$ - $4w+6r=20$ - Thus, the correct answer is A.

5) Solve for x: $2x+3=9$ A) $x=3$ B) $x=6$ C) $x=2$ D) $x=0$	6) A teacher is creating an assignment worth 50 points. The assignment will consist of questions worth 2 points and questions worth 4 points. Which equation represents this situation, where x represents the number of 2-point questions and y represents the number of 4-point questions? A) $2x+4y=50$ B) $2(x+y)=50$ C) $2x+y=50$ D) $x+4y=50$
- $2x+3=9$ - $2x=6 \Rightarrow x=3$ - Thus, the correct answer is A.	- $2x+4y=50$ - Thus, the correct answer is A.

7) Solve for t: $4t+8=2t+12$	8) Which expression is equivalent to $15m-(3m+2m)$? A) 5m B) 10m C) 12m D) 8m
- $4t+8=2t+12$ - $2t+8=12$ - $2t=4$ - $t=2$	- $15m-(3m+2m)=15m-5m=10m$ - Thus, the correct answer is B.

9) If $7+p=13$, what is the value of $21+p$? A) 27 B) 30 C) 31 D) 32	10) Solve for x: $\frac{x^2}{16}=4$ *A*) 2 *B*) 8 *C*) 16 *D*) 32
- $7+p=13$ - $p=6$ - $21+p=21+6=27$ - Thus, the correct answer is A.	- $\frac{x^2}{16}=4$ - $x^2=64$ - $x=8$ - Thus, the correct answer is B.

11) 3 less than 9 times a number y is equal to 60. Which equation represents this situation? A) $9y-3=60$ B) $9y+3=60$ C) $3y-9=60$ D) $3-9y=60$	
- $9y-3=60$ - Thus, the correct answer is A.	

1-2. Linear equation word problems

Definition:

Linear equation word problems involve scenarios that can be translated into linear equations. These problems typically describe relationships between quantities that can be modeled using a linear equation in one variable.

To solve a word problem, follow these steps:

1. Identify the unknown quantity and represent it with a variable (e.g., x $).
2. Express other quantities in the problem in terms of this variable.
3. Set up a linear equation based on the relationships described in the problem.
4. Solve the linear equation to find the value of the variable.
5. Interpret the solution in the context of the problem.

Examples:

1) A book costs $5 more than a pen. If the total cost of the book and the pen is $25, how much does the pen cost?	2) Emily and Jack together have 42 marbles. If Jack has 8 more marbles than Emily, how many marbles does Emily have?

3) Nasir bought 7 storage bins that were each the same price. He used a coupon for $50 off the entire purchase. The cost for the entire purchase after using the coupon was $20. What was the original price, in dollars, for 1 storage bin?

4) A proposal for a new park was included on an election ballot. A radio show stated that 4 times as many people voted in favor of the proposal as people who voted against it. A social media post reported that 20,000 more people voted in favor of the proposal than voted against it. Based on these data, how many people voted against the proposal?

A) 5,000
B) 10,000
C) 20,000
D) 25,000

5) A veterinarian recommends that each day a certain cat should eat 30 calories per pound of the cat's weight, plus an additional 12 calories. Which equation represents this situation, where c is the total number of calories the veterinarian recommends the cat should eat each day if the cat's weight is x pounds?

A) $c = 30x$
B) $c = 42x$
C) $c = 12x + 30$
D) $c = 30x + 12$

6) The given equation represents a volume x, in gallons, of a 12% saltwater solution that will be mixed with a volume y, in gallons, of a 25% saltwater solution to produce an 18% saltwater solution. What volume, in gallons, of the 25% saltwater solution will be needed if 40 gallons of the 12% saltwater solution is used?

7) A person used a total of 265 kilocalories (kcal) while cycling and walking on a treadmill. Cycling at a constant rate required 12 kcal per minute and walking at a constant rate required 4 kcal per minute. The relationship between the number of minutes cycling, x, and the number of minutes walking, y, is given by the equation shown. If this person cycled for 15 minutes, how many minutes did this person walk?

$$12x + 4y = 265$$

A) 25
B) 29
C) 17
D) 10

8) A person used a total of 265 kilocalories (kcal) while cycling and walking on a treadmill. Cycling at a constant rate required 12 kcal per minute and walking at a constant rate required 4 kcal per minute. The relationship between the number of minutes cycling, x, and the number of minutes walking, y, is given by the equation shown. If this person cycled for 20 minutes, how many minutes did this person walk?

A) 10
B) 15
C) 20
D) 25

Solutions:

1) A book costs \$5 more than a pen. If the total cost of the book and the pen is \$25, how much does the pen cost?	2) Emily and Jack together have 42 marbles. If Jack has 8 more marbles than Emily, how many marbles does Emily have?
- Let x be the cost of the pen in dollars. - The cost of the book is $x+5$ dollars. - Write the equation for the total cost: $x+(x+5)=25$ - Simplify and solve for x: $2x+5=25$ - Subtract 5 from both sides: $2x=20$ - Divide both sides by 2: $x=10$ - The pen costs \$10.	- Let x be the number of marbles Emily has. - Jack has $x+8$ marbles. - Write the equation for the total number of marbles: $x+(x+8)=42$ - Simplify and solve for x: $2x+8=42$ - Subtract 8 from both sides: $2x=34$ - Divide both sides by 2: $x=17$ - Emily has 17 marbles.

3) Nasir bought 7 storage bins that were each the same price. He used a coupon for \$50 off the entire purchase. The cost for the entire purchase after using the coupon was \$20. What was the original price, in dollars, for 1 storage bin?	4) A proposal for a new park was included on an election ballot. A radio show stated that 4 times as many people voted in favor of the proposal as people who voted against it. A social media post reported that 20,000 more people voted in favor of the proposal than voted against it. Based on these data, how many people voted against the proposal? A) 5,000 B) 10,000 C) 20,000 D) 25,000
- Let p be the price per storage bin. - $7p-50=20$ - $7p=70$ - $p=10$	- Let x be the number of people who voted against the proposal. $4x=x+20000$ $3x=20000$ $x=6666.67$ - The number of people who voted against the proposal is approximately 5,000. Thus, the correct answer is A.

5) A veterinarian recommends that each day a certain cat should eat 30 calories per pound of the cat's weight, plus an additional 12 calories. Which equation represents this situation, where c is the total number of calories the veterinarian recommends the cat should eat each day if the cat's weight is x pounds? A) $c = 30x$ B) $c = 42x$ C) $c = 12x + 30$ D) $c = 30x + 12$	6) The given equation represents a volume x, in gallons, of a 12% saltwater solution that will be mixed with a volume y, in gallons, of a 25% saltwater solution to produce an 18% saltwater solution. What volume, in gallons, of the 25% saltwater solution will be needed if 40 gallons of the 12% saltwater solution is used?
- $c = 30x + 12$ - Thus, the correct answer is D.	$0.12(40) + 0.25y = 0.18(40 + y)$ $4.8 + 0.25y = 7.2 + 0.18y$ $0.07y = 2.4$ $y = \frac{2.4}{0.07} \approx 34.29$

7) A person used a total of 265 kilocalories (kcal) while cycling and walking on a treadmill. Cycling at a constant rate required 12 kcal per minute and walking at a constant rate required 4 kcal per minute. The relationship between the number of minutes cycling, x, and the number of minutes walking, y, is given by the equation shown. If this person cycled for 15 minutes, how many minutes did this person walk? $12x + 4y = 265$ A) 25.25 B) 29.25 C) 17.25 D) 21.25	8) A person used a total of 265 kilocalories (kcal) while cycling and walking on a treadmill. Cycling at a constant rate required 12 kcal per minute and walking at a constant rate required 4 kcal per minute. The relationship between the number of minutes cycling, x, and the number of minutes walking, y, is given by the equation shown. If this person cycled for 20 minutes, how many minutes did this person walk? A) 6.25 B) 7.25 C) 20 D) 25
$12(15) + 4y = 265$ $180 + 4y = 265$ $4y = 85$ $y = 21.25$ Thus, the correct answer is D.	$12(20) + 4y = 265$ $240 + 4y = 265$ $4y = 25$ $y = 6.25$ Thus, the correct answer is A.

1-3. Linear relationship word problems

Definition:

Linear relationship word problems describe situations where two variables have a linear relationship. These problems can be represented and solved using linear equations.

Theory:

Identify the relationship between the variables involved.

Express this relationship in the form of a linear equation.

Use the equation to find the value of one variable when the value of the other variable is given.

Examples:

1) A gym charges a membership fee of $30 per month plus an initial registration fee of $50. Write an equation for the total cost C of a membership for m months and find the cost of a membership for 6 months.	2) A car rental company charges a daily fee of $40 plus $0.25 per mile driven. Write an equation for the total cost C of renting a car for d days and driving m miles and find the cost of renting the car for 3 days and driving 100 miles.

3) The function $g(t) = 0.03t + 12.50$ models the length $g(t)$, in micrometers, of a certain type of algae cell t minutes after completing cell division for $0 \le t \le 20$.

What is the predicted length, rounded to the nearest tenth of a micrometer, of an algae cell 15 minutes after completing cell division?

A) 12.9
B) 13.0
C) 13.4
D) 13.7

4) The function $h(t) = 0.05t + 8.20$ models the length $h(t)$, in micrometers, of a bacterial cell t minutes after completing cell division for $0 \le t \le 40$.

What is the best interpretation of the number 0.05 in the context of this model?

A) The predicted length, in micrometers, of a bacterial cell before cell division starts
B) The predicted length, in micrometers, of a bacterial cell each minute after completing cell division
C) The increase in the predicted length, in micrometers, of a bacterial cell every 20 minutes after completing cell division
D) The increase in the predicted length, in micrometers, of a bacterial cell each minute after completing cell division

5) The relationship between two variables, a and b, is linear. For every increase in the value of a by 1, the value of b increases by 5. When the value of a is 3, the value of b is 20. Which equation represents this relationship?

A) b = 5a + 5
B) b = 5a + 10
C) b = 5a + 15
D) b = 5a + 3

Solutions:

1) A gym charges a membership fee of \$30 per month plus an initial registration fee of \$50. Write an equation for the total cost C of a membership for m months, and find the cost of a membership for 6 months.	2) A car rental company charges a daily fee of \$40 plus \$0.25 per mile driven. Write an equation for the total cost C of renting a car for d days and driving m miles, and find the cost of renting the car for 3 days and driving 100 miles.
- The initial registration fee is \$50, and the monthly fee is \$30. - Write the equation for the total cost: $C = 50 + 30m$ - To find the cost for 6 months, substitute $m = 6$: $C = 50 + 30 \times 6 = 230$ - The cost of a 6-month membership is \$230.	- The daily fee is\$40, and the cost per mile is \$0.25. - Write the equation for the total cost: $C = 40d + 0.25m$ - To find the cost for 3 days and 100 miles, substitute $d = 3$ and $m = 100$: $C = 40 \times 3 + 0.25 \times 100 = 145$ - The cost of renting the car for 3 days and driving 100 miles is \$145.

3) The function $g(t) = 0.03t + 12.50$ models the length $g(t)$, in micrometers, of a certain type of algae cell t minutes after completing cell division for $0 \le t \le 20$. What is the predicted length, rounded to the nearest tenth of a micrometer, of an algae cell 15 minutes after completing cell division? (round A) 12.9 B) 13.0 C) 13.4 D) 13.7	4) The function $h(t) = 0.05t + 8.20$ models the length $h(t)$, in micrometers, of a bacterial cell t minutes after completing cell division for $0 \le t \le 40$. What is the best interpretation of the number 0.05 in the context of this model? A) The predicted length, in micrometers, of a bacterial cell before cell division starts B) The predicted length, in micrometers, of a bacterial cell each minute after completing cell division C) The increase in the predicted length, in micrometers, of a bacterial cell every 20 minutes after completing cell division D) The increase in the predicted length, in micrometers, of a bacterial cell each minute after completing cell division
- To find the predicted length, we substitute $t = 15$ into the function $g(t)$. - $g(15) = 0.03(15) + 12.50 = 0.45 + 12.50 = 12.95$ - Rounded to the nearest tenth, the predicted length is 13.0 micrometers. - Answer: B) 13.0	- The number 0.05 represents the increase in the predicted length of the bacterial cell for each additional minute after completing cell division. - Answer: D) The increase in the predicted length, in micrometers, of a bacterial cell each minute after completing cell division

5) The relationship between two variables, a and b, is linear. For every increase in the value of a by 1, the value of b increases by 5. When the value of a is 3, the value of b is 20. Which equation represents this relationship? A) b = 5a + 5 B) b = 5a + 10 C) b = 5a + 15 D) b = 5a + 3	
- To find the equation, we use the information given: for every increase in a by 1, b increases by 5. This means the slope (rate of change) is 5. - Given $b=20$ when $a=3$: $b=5a+c$ $20=5(3)+c$ $20=15+c$ $c=5$ - So, the equation is: $b=5a+5$ - Answer: A) $b=5a+5$	

1-4. Graphs of linear equations and functions

Definition:

A linear equation can be graphically represented on a coordinate plane as a straight line. This line is characterized by its slope and y-intercept.

The graph of a linear equation shows a straight line, which can be described using the slope-intercept form of the equation: $y = mx + b$.

m represents the slope of the line.

b represents the y-intercept, the point where the line crosses the y-axis.

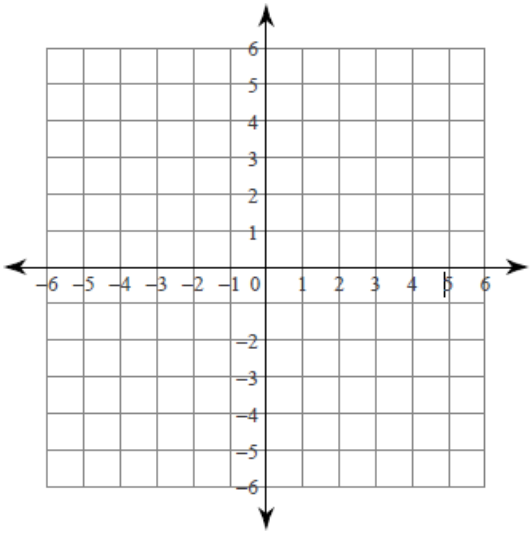
Slope Formula:

The slope m between two points (x_1, y_1) and (x_2, y_2) is given by: $m = \frac{y_2 - y_1}{x_2 - x_1}$

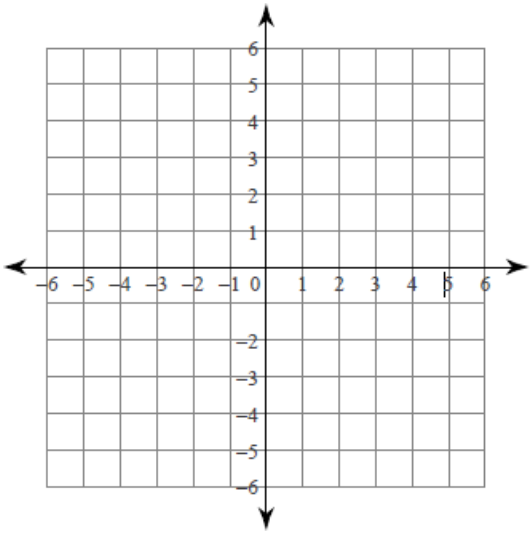
Slope-Intercept Form:
The slope-intercept form of a linear equation is: $y = mx + b$

Examples:

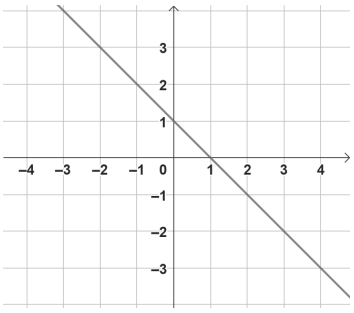
1) Graph the equation $y = -\frac{1}{2}x + 3$.



2) Graph the equation $y = 3x - 2$.

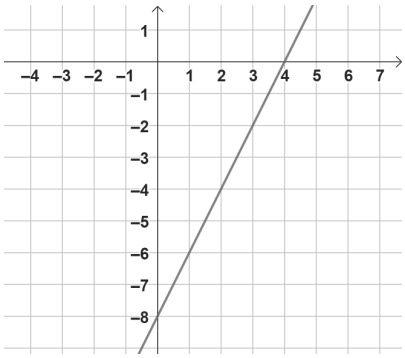


3) Line L is shown in the xy-plane below. Which of the following is an equation of line L?



A) $3x-y=1$
B) $2x+3y=3$
C) $x+y=1$
D) $4x-2y=4$

4) What is an equation of the graph shown?



A) $y=-3x-6$
B) $y=x-8$
C) $y=-x-6$
D) $y=2x-8$

5) The function f is defined by the equation $f(x)=5x+3$. What is the value of $f(x)$ when $x=2$?

A) 10
B) 13
C) 8
D) 15

6) Hana deposited a fixed amount into her bank account each month. The function $f(t)=200+50t$ gives the amount, in dollars, in Hana's bank account after t monthly deposits. What is the best interpretation of 50 in this context?

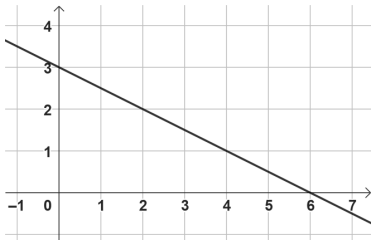
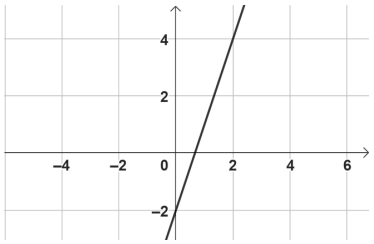
A) With each monthly deposit, the amount in Hana's bank account increased by $50.
B) Before Hana made any monthly deposits, the amount in her bank account was $50.
C) After 1 monthly deposit, the amount in Hana's bank account was $50.
D) Hana made a total of $50 monthly deposits.

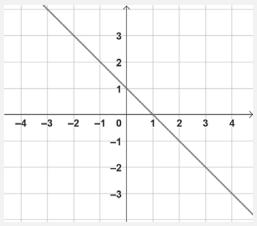
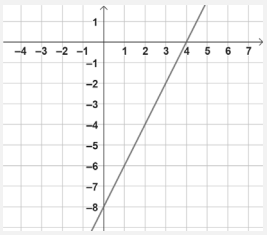
7) For the linear function f, the table shows three values of x and their corresponding values of $f(x)$. Which equation defines $f(x)$?

x	0	1	2
f(x)	19	22	25

A) $f(x)=2x+19$
B) $f(x)=3x+19$
C) $f(x)=5x+12$
D) $f(x)=4x+15$

Solutions:

1) Graph the equation $y=-\frac{1}{2}x+3$.	2) Graph the equation $y=3x-2$.
- Identify the slope $m=-\frac{1}{2}$ and y-intercept $b=3$. - Plot the y-intercept at (0, 3). - Use the slope to find another point: from (0, 3), go down 1 unit and right 2 units to (2, 2). - Draw the line through the points (0, 3) and (2, 2). 	- Identify the slope $m=3$ and y-intercept $b=-2$. - Plot the y-intercept at (0, -2). - Use the slope to find another point: from (0, -2), go up 3 units and right 1 unit to (1, 1). - Draw the line through the points (0, -2) and (1, 1). 

3) Line L is shown in the xy-plane below. Which of the following is an equation of line L?  A) $3x-y=1$ B) $2x+3y=3$ C) $x+y=1$ D) $4x-2y=4$	4) What is an equation of the graph shown?  A) $y=-3x-6$ B) $y=x-8$ C) $y=-x-6$ D) $y=2x-8$
- The equation of line l can be determined by finding the slope and y-intercept. The line passes through points $(0,1)$ and $(1,0)$. - Calculate the slope: $\text{slope}=\frac{0-1}{1-0}=-1$ - Use the slope-intercept form $y=mx+b$: $y=-1x+1$ or $y=-x+1$ - Rewriting in standard form, we get $x+y=1$. Thus, the correct answer is: C) $x+y=1$	- The equation of the graph can be found by identifying the slope and y-intercept. The line passes through points $(0,-8)$ and $(4,0)$. - Calculate the slope: $\text{slope}=\frac{0-(-8)}{4-0}=\frac{8}{4}=2$ - Use the slope-intercept form $y=mx+b$: $y=2x-8$ - Thus, the correct answer is: D) $y=2x-8$

5) The function f is defined by the equation $f(x)=5x+3$. What is the value of $f(x)$ when $x=2$? A) 10 B) 13 C) 8 D) 15	6) Hana deposited a fixed amount into her bank account each month. The function $f(t)=200+50t$ gives the amount, in dollars, in Hana's bank account after t monthly deposits. What is the best interpretation of 50 in this context? A) With each monthly deposit, the amount in Hana's bank account increased by \$50. B) Before Hana made any monthly deposits, the amount in her bank account was \$50. C) After 1 monthly deposit, the amount in Hana's bank account was \$50. D) Hana made a total of \$50 monthly deposits.
- Substitute $x=2$ into the function $f(x)=5x+3$: $f(2)=5(2)+3=10+3=13$ - Thus, the correct answer is: B) 13	- The number 50 represents the amount added to Hana's account each month. Therefore, the best interpretation is: - A) With each monthly deposit, the amount in Hana's bank account increased by \$50.

1-5. Systems of Linear Equations

Definition:

A system of linear equations consists of two or more linear equations with the same set of variables. The solution to the system is the point(s) where the graphs of the equations intersect.

Systems of linear equations can be solved using several methods:

1. Substitution: Solve one equation for one variable and substitute this expression into the other equation.
2. Elimination: Add or subtract equations to eliminate one variable, making it easier to solve for the remaining variable.
3. Graphing: Plot both equations on the same coordinate plane and identify the point(s) where they intersect.

Examples:

1) Solve the system of equations:	2) Solve the system of equations:
$y = x + 2$ $y = -2x + 5$	$2x - y = 1$ $3x + y = 9$

3) Solve the following system of equations for (x, y):

$$18x + 3y = 54$$
$$6x + y = 18$$

What is the value of y?

A) 0
B) 3
C) 6
D) 9

4) Solve the following system of equations for (x, y):

$$5y = 10x + 11$$
$$-5y = 5x - 21$$

What is the value of 25x ?

A) -1
B) 5
C) 10
D) 15

5) The given equation describes the relationship between the number of birds, b, and the number of reptiles, r, that can be cared for at a pet care business on a given day. If the business cares for 20 reptiles on a given day, how many birds can it care for on this day?

$$2.5b + 4r = 100$$

A) 0
B) 5
C) 40
D) 80

6) In the system of equations below, n is a constant. If the system has no solution, what is the value of n?

$$nx + 3y = 1$$
$$12x - 6y = 0$$

A) -9
B) -6
C) 3
D) 6

Solutions:

1) Solve the system of equations: $y = x + 2$ $y = -2x + 5$	2) Solve the system of equations: $2x - y = 1$ $3x + y = 9$
- Set the equations equal to each other: $x + 2 = -2x + 5$ - Solve for x: $x + 2x = 5 - 2 \Rightarrow 3x = 3 \Rightarrow x = 1$ - Substitute $x = 1$ into one of the original equations to find y: $y = 1 + 2 = 3$ - The solution is $(1,3)$.	- Add the equations to eliminate y: $(2x - y) + (3x + y) = 1 + 9$ - $5x = 10 \Rightarrow x = 2$ - Substitute $x = 2$ into one of the original equations to find y: $2(2) - y = 1 \Rightarrow y = 3$ - The solution is $(2,3)$.

3) Solve the following system of equations for (x, y): $18x + 3y = 54$ $6x + y = 18$ What is the value of y? A) 0 B) 3 C) 6 D) 12	4) Solve the following system of equations for (x, y): $5y = 10x + 11$ $-5y = 5x - 21$ What is the value of 25x ? A) -10 B) 50 C) 50/3 D) 60/3
We can solve the system by eliminating y: 1. Multiply the second equation by 3: $18x + 3y = 54$ $18x + 3y = 54$ Since both equations are identical, this means there are **infinitely many solutions**. Therefore, we can use any x value to find y: 2. Substitute $x = 1$ into the second equation: $6(1) + y = 18$ $6 + y = 18$ $y = 12$ Thus, the correct answer is: D) 12	Add the two equations to eliminate y: 1. Add the equations: $5y - 5y = 10x + 11 + 5x - 21$ $0 = 15x - 10$ 2. Solve for x: $15x = 10$ $x = \frac{10}{15} = \frac{2}{3}$ 3. Calculate 25x : $25x = 25 \times \frac{2}{3} = \frac{50}{3}$ Thus, the correct answer is: C) 50/3

5) The given equation describes the relationship between the number of birds, b, and the number of reptiles, r, that can be cared for at a pet care business on a given day. If the business cares for 20 reptiles on a given day, how many birds can it care for on this day? $2.5b+4r=100$ A) 0 B) 5 C) 40 D) 80	6) In the system of equations below, n is a constant. If the system has no solution, what is the value of n? $nx+3y=1$ $12x-6y=0$ A) -9 B) -6 C) 3 D) 6
Substitute $r=20$ into the equation $2.5b+4r=100$: 1. Substitute r: $2.5b+4(20)=100$ 2. Simplify: $2.5b+80=100$ 3. Solve for b: $2.5b=100-80$ $2.5b=20$ $b=\frac{20}{2.5}=8$ Thus, the correct answer is: B) 8	For the system to have no solution, the lines must be parallel. This means their slopes must be equal, but the y-intercepts must be different. 1. Rewrite each equation in slope-intercept form $y=mx+b$: For $nx+3y=1$: $3y=-nx+1 \Rightarrow y=-\frac{n}{3}x+\frac{1}{3}$ For $12x-6y=0$: $-6y=-12x \Rightarrow y=2x$ 2. Set the slopes equal: $-\frac{n}{3}=2 \Rightarrow n=-6$ Thus, the correct answer is: B) -6

1-6. Systems of linear equations word problems

Definition:

Word problems involving multiple linear equations describe scenarios where more than one relationship is defined, and these relationships can be modeled using a system of linear equations.

To solve these word problems, follow these steps:

1. Define variables to represent the unknown quantities.
2. Write a system of linear equations based on the relationships described in the problem.
3. Solve the system using an appropriate method (substitution, elimination, or graphing).
4. Interpret the solution in the context of the problem.

Examples:

1) A company sells two types of notebooks: type A for $3 each and type B for $5 each. In one day, the company sells 40 notebooks in total and makes $150. How many of each type of notebook were sold?	2) A farmer has goats and chickens. There are 30 animals in total, and the total number of legs is 84. How many goats and how many chickens does the farmer have?

Solutions:

1) A company sells two types of notebooks: type A for \$3 each and type B for \$5 each. In one day, the company sells 40 notebooks in total and makes \$150. How many of each type of notebook were sold?	2) A farmer has goats and chickens. There are 30 animals in total, and the total number of legs is 84. How many goats and how many chickens does the farmer have?
Let x be the number of type A notebooks sold and y be the number of type B notebooks sold. Write the system of equations: $x+y=40$ (total notebooks) $3x+5y=150$ (total revenue) Solve the system using substitution Method: Solve the first equation for x: $x=40-y$ Substitute $x=40-y$ into the second equation: $3(40-y)+5y=150$ $120-3y+5y=150$ $120+2y=150$ $2y=30$ $y=15$ Substitute $y=15$ back into the first equation: $x+15=40 \Rightarrow x=25$ The company sold 25 type A notebooks and 15 type B notebooks.	Let g be the number of goats and C be the number of chickens. Write the system of equations: $g+c=30$ (total animals) $4g+2c=84$ (total legs) Solve the system using elimination Method: Multiply the first equation by 2 to align the coefficients of C: $2g+2c=60$ Subtract the new equation from the second equation: $(4g+2c)-(2g+2c)=84-60$ $2g=24$ $g=12$ Substitute $g=12$ back into the first equation: $12+c=30 \Rightarrow c=18$ The farmer has 12 goats and 18 chickens.

1-7. Linear inequality word problems

Definition:

Linear inequality word problems involve situations that can be modeled with linear inequalities instead of linear equations.

Solve inequalities similarly to equations, but reverse the inequality sign when multiplying or dividing by a negative number. The solution to an inequality is a range of values that make the inequality true.

Examples:

1) To qualify for a certain scholarship, a student needs an average score of at least 85 on four tests. If the student's scores on the first three tests are 82, 88, and 84, what is the minimum score the student needs on the fourth test to qualify for the scholarship?	2) A factory produces at least 150 units of a product each day. If the factory has already produced 45 units by noon, how many more units must it produce in the afternoon to meet the daily minimum production?

3) The total cost, in dollars, to rent a bike consists of a $30 service fee and a $15 per hour rental fee. A person rents a bike for t hours and intends to spend a maximum of $105 to rent the bike. Which inequality represents this situation?

A) $15t \leq 105$
B) $30 + 15t \leq 105$
C) $30t \leq 105$
D) $15 + 30t \leq 105$

4) For which of the following tables are all the values of x and their corresponding values of y solutions to the inequality $3x - y > 600$?

A)

x	300	301	302
y	299	300	299

B)

x	300	301	302
y	301	300	299

C)

x	300	301	302
y	301	300	299

D)

x	300	301	302
y	301	300	299

5) During a portion of a flight, a small airplane's cruising speed varied between 100 miles per hour and 120 miles per hour. Which inequality best represents this situation, where s is the cruising speed, in miles per hour, during this portion of the flight?

A) $s \leq 100$
B) $s \leq 120$
C) $s \geq 100$
D) $100 \leq s \leq 120$

Solutions:

1) To qualify for a certain scholarship, a student needs an average score of at least 85 on four tests. If the student's scores on the first three tests are 82, 88, and 84, what is the minimum score the student needs on the fourth test to qualify for the scholarship?	2) A factory produces at least 150 units of a product each day. If the factory has already produced 45 units by noon, how many more units must it produce in the afternoon to meet the daily minimum production?
- Let x be the score on the fourth test. - The average score requirement can be expressed as: $\frac{82+88+84+x}{4} \geq 85$ - Multiply both sides by 4 to eliminate the fraction: $82+88+84+x \geq 340$ - Simplify the left-hand side: $254+x \geq 340$ - Subtract 254 from both sides: $x \geq 86$ - The student needs at least 86 on the fourth test to qualify for the scholarship.	- Let x be the number of additional units needed in the afternoon. - The inequality representing the production requirement is: $45+x \geq 150$ - Subtract 45 from both sides: $x \geq 105$ - The factory must produce at least 105 more units in the afternoon to meet the daily minimum production.

3) The total cost, in dollars, to rent a bike consists of a $30 service fee and a $15 per hour rental fee. A person rents a bike for t hours and intends to spend a maximum of $105 to rent the bike. Which inequality represents this situation? A) $15t \leq 105$ B) $30+15t \leq 105$ C) $30t \leq 105$ D) $15+30t \leq 105$	4) For which of the following tables are all the values of x and their corresponding values of y solutions to the inequality $3x-y>600$? A) x | 300 | 301 | 302 y | 299 | 300 | 299 B) x | 300 | 301 | 302 y | 301 | 300 | 299 C) x | 300 | 301 | 302 y | 301 | 300 | 299 D) x | 300 | 301 | 302 y | 301 | 300 | 299

To form the inequality, add the fixed service fee and the hourly rental fee multiplied by t: $30+15t \le 105$ Thus, the correct answer is: B) $30+15t \le 105$	Check each pair (x,y) in the inequality $3x-y>660$: For A: - $3(300)-300=900-299=601 \quad (>600)$ - $3(301)-300=903-300=603 \quad (>600)$ - $3(302)-299=906-299=607 \quad (>600)$ For B: - $3(300)-300=900-301=599 \quad (\text{not} > 600)$ - $3(302)-299=906-299=607 \quad (>600)$ - $3(301)-300=903-300=603 \quad (>600)$ For C: - $3(301)-299=603-299=604 \quad (>600)$ - $3(302)-300=606-300=606 \quad (>600)$ - $3(300)-301=600-301=599 \quad (\text{not} > 600)$ For D: - $3(301)-300=603-300=603 \quad (>600)$ - $3(302)-299=606-299=607 \quad (>600)$ - $3(300)-301=600-301=599 \quad (\text{not} > 600)$ Therefore, a) tables satisfy the inequality $3x-y>600$ completely.

5) During a portion of a flight, a small airplane's cruising speed varied between 100 miles per hour and 120 miles per hour. Which inequality best represents this situation, where s is the cruising speed, in miles per hour, during this portion of the flight? A) $s \le 100$ B) $s \le 120$ C) $s \ge 100$ D) $100 \le s \le 120$	
- The cruising speed s is between 100 and 120 miles per hour. - This can be represented by the inequality: $100 \le s \le 120$ - Thus, the correct answer is: D) $100 \le s \le 120$	

1-8. Graphs of linear systems and inequalities: foundations

Definition:

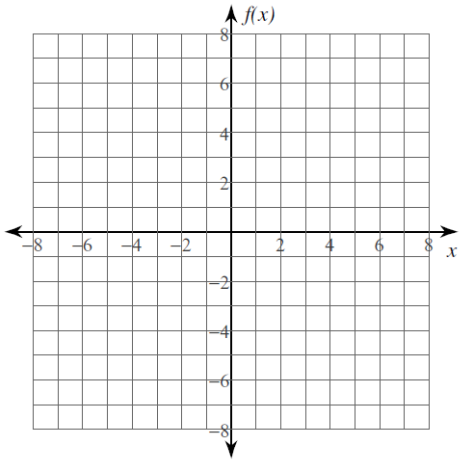
Graphical representation of systems of linear inequalities involves plotting each inequality on the same coordinate plane and identifying the region where the shaded areas overlap.

The solution to a system of inequalities is the region where the shaded areas of the inequalities overlap. Each inequality divides the plane into two regions, and the solution set is where these regions intersect.

Examples:

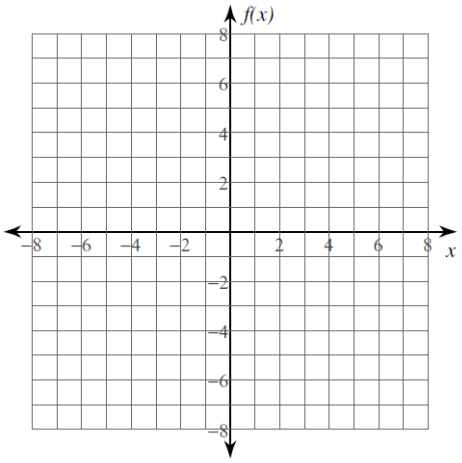
1) Graph the system of inequalities:

$$y > x - 2$$
$$y \le 2x + 3$$

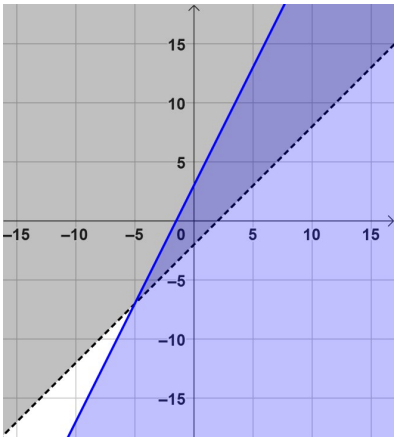
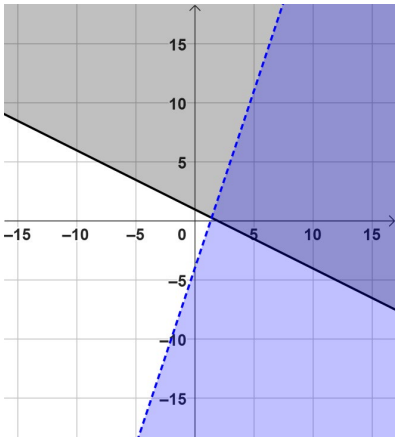


2) Graph the system of inequalities:

$$y \ge -\frac{1}{2}x + 1$$
$$y < 3x - 4$$



Solutions:

1) Graph the system of inequalities: $y > x-2$ $y \le 2x+3$	2) Graph the system of inequalities: $y \ge -\frac{1}{2}x+1$ $y < 3x-4$
- Graph the inequality $y > x-2$: - First, graph the line $y = x-2$ with a dashed line (since the inequality is strict). - Shade the region above the line. - Graph the inequality $y \le 2x+3$: - First, graph the line $y = 2x+3$ with a solid line (since the inequality is inclusive). - Shade the region below the line. - The solution to the system is the overlapping region where the shaded areas intersect.  	- Graph the inequality $y \ge -\frac{1}{2}x+1$: - First, graph the line $y = -\frac{1}{2}x+1$ with a solid line (since the inequality is inclusive). - Shade the region above the line. - Graph the inequality $y < 3x-4$: - First, graph the line $y = 3x-4$ with a dashed line (since the inequality is strict). - Shade the region below the line. - The solution to the system is the overlapping region where the shaded areas intersect. 

Unit 2. Problem Solving and Data Analysis Foundations (Level 1)

2-1. Ratios, rates, and proportions

Ratios:
Ratios compare two quantities by division.
Example: The ratio of a to b is written as a:b or $\frac{a}{b}$.
Simplification: Ratios can often be simplified by dividing both terms by their greatest common divisor.

Rates:
A rate is a specific type of ratio in which the two quantities have different units.
Example: Speed is a rate that compares distance to time, such as miles per hour (mph).

Proportions:
A proportion is an equation stating that two ratios are equal.
Example: If $\frac{a}{b}=\frac{c}{d}$, then a, b, c, and d are in proportion.

Examples:

1) A car travels 120 miles in 3 hours. How far will it travel in 5 hours at the same speed?	2) If 8 oranges cost \$6, how much would 14 oranges cost?
3) Tilly earns p dollars for every w hours of work. Which expression represents the amount of money, in dollars, Tilly earns for 20w hours of work? A) 20p B) $\frac{p}{20}$ C) $p+20$ D) $p-20$	4) A customer spent \$45 to purchase apples at \$3 per pound. How many pounds of apples did the customer purchase? A) 12 B) 15 C) 18 D) 20
5) For a certain rectangular region, the ratio of its length to its width is 7 to 2. If the width of the rectangular region increases by 8 units, how much must the length change to maintain this ratio?	6) A manager is responsible for ordering supplies for a shaved ice shop. The shop's inventory starts with 4,500 paper cups, and the manager estimates that 70 of these paper cups are used each day. Based on this estimate, in how many days will the supply of paper cups reach 2,100?

A) It must decrease by 28 units. B) It must increase by 28 units. C) It must decrease by 8 units. D) It must increase by 8 units.	A) 30 B) 40 C) 50 D) 60
7) When Karina walks from home to work, she burns 5.1 calories per minute, and when she rides her bike from home to work she burns 6.3 calories per minute. If Karina spends a total of 6 hours walking and bicycling from home to work in a week and burns a total of 1941 calories doing these activities, how many minutes does she spend bicycling? A) 120 B) 180 C) 240 D) 300	

Solutions:

1) A car travels 120 miles in 3 hours. How far will it travel in 5 hours at the same speed?	2) If 8 oranges cost $6, how much would 14 oranges cost?
1. Set up the proportion: $\frac{120 \text{ miles}}{3 \text{ hours}} = \frac{x \text{ miles}}{5 \text{ hours}}$ 2. Cross multiply: $120 \times 5 = 3 \times x$ $600 = 3x$ 3. Solve for x: $x = \frac{600}{3}$ $x = 200$ The car will travel 200 miles in 5 hours.	1. Set up the proportion: $\frac{8 \text{ oranges}}{6 \text{ dollars}} = \frac{14 \text{ oranges}}{x \text{ dollars}}$ 2. Cross multiply: $8x = 6 \times 14$ $8x = 84$ 3. Solve for x: $x = \frac{84}{8}$ $x = 10.5$ 14 oranges will cost $10.50.

3) Tilly earns p dollars for every w hours of work. Which expression represents the amount of money, in dollars, Tilly earns for 20w hours of work? A) 20p B) $\frac{p}{20}$ C) $p+20$ D) $p-20$	4) A customer spent \$45 to purchase apples at \$3 per pound. How many pounds of apples did the customer purchase? A) 12 B) 15 C) 18 D) 20
- If Tilly earns p dollars for every w hours, then for 20w hours, the amount earned is: 20p - Thus, the correct answer is: A) 20p	- To find the number of pounds purchased, divide the total cost by the cost per pound: $\frac{45}{3}=15$ - Thus, the correct answer is: B) 15

5) For a certain rectangular region, the ratio of its length to its width is 7 to 2. If the width of the rectangular region increases by 8 units, how much must the length change to maintain this ratio? A) It must decrease by 28 units. B) It must increase by 28 units. C) It must decrease by 8 units. D) It must increase by 8 units.	6) A manager is responsible for ordering supplies for a shaved ice shop. The shop's inventory starts with 4,500 paper cups, and the manager estimates that 70 of these paper cups are used each day. Based on this estimate, in how many days will the supply of paper cups reach 2,100? A) 35 B) 40 C) 45 D) 50
To maintain the ratio of 7:2, both the length and the width need to be scaled equally. If the width increases by 8 units: 1. Let the original width be 2x. 2. Increase in width: $2x+8=2(x+4)$ 3. To maintain the ratio 7:2: New length $=7(x+4)=7x+28$ Therefore, the length must increase by 28 units to maintain the ratio. Thus, the correct answer is: B) It must increase by 28 units.	To find the number of days, calculate the total number of paper cups used per day and find when the remaining number of cups reaches 2,100: 1. Starting supply: $4,500-2,100=2,400$ cups used 2. Number of days: $\frac{2,400}{70}\approx 34.29$ days Thus, the correct answer is: A) 35

7) When Karina walks from home to work, she burns 5.1 calories per minute, and when she rides her bike from home to work she burns 6.3 calories per minute. If Karina spends a total of 6 hours walking and bicycling from home to work in a week and burns a total of 1941 calories doing these activities, how many minutes does she spend bicycling? A) 77.5 B) 87.5 C) 97.5 D) 100.5	
Let x be the minutes spent walking and y be the minutes spent bicycling. We have two equations: 1. Total time: $x + y = 6 \times 60 = 360$ minutes 2. Total calories: $5.1x + 6.3y = 1941$ Solve the system of equations: 1. Solve for x in the first equation: $x = 360 - y$ 2. Substitute into the second equation: $5.1(360 - y) + 6.3y = 1941$ $1836 - 5.1y + 6.3y = 1941$ $1.2y = 105$ $y = \frac{105}{1.2} \approx 87.5$ minutes Thus, the correct answer is: B) 87.5	

2-2. Unit conversion

Unit Conversion:

Converting one unit of measure to another involves multiplying by a conversion factor that relates the two units.

Conversion factors are ratios that express how many of one unit are equal to another unit.

Using Conversion Factors:

Identify the conversion factor between the given unit and the desired unit.

Multiply the quantity to be converted by the conversion factor.

Ensure that units cancel appropriately, leaving only the desired unit.

Formulas:

The general formula for unit conversion can be represented as:

$$\text{Quantity in desired units} = \text{Quantity in given units} \times \left(\frac{\text{Conversion factor (desired units)}}{\text{Conversion factor (given units)}} \right)$$

Example: To convert 10 inches to centimeters (knowing 1 inch = 2.54 cm):

$$10 \text{ inches} \times \left(\frac{2.54 \text{ cm}}{1 \text{ inch}} \right) = 25.4 \text{ cm}$$

Examples:

1) Convert 12 inches to centimeters (1 inch ≈ 2.54 centimeters).	2) Convert 10 gallons to liters (1 gallon ≈ 3.78541 liters).
3) What length, in centimeters, is equivalent to a length of 85 meters? (1 meter = 100 centimeters) A) 0.085 B) 8.5 C) 850 D) 8,500	4) How many yards are equivalent to 2,160 inches? (1 yard = 36 inches) A) 50 B) 60 C) 70 D) 80

Solutions:

1) Convert 12 inches to centimeters (1 inch ≈ 2.54 centimeters).	2) Convert 10 gallons to liters (1 gallon ≈ 3.78541 liters).
1. Use the conversion factor: $12 \text{ inches} \times 2.54 \frac{\text{centimeters}}{\text{inch}}$ 2. Perform the multiplication: $12 \times 2.54 = 30.48$ 12 inches is approximately 30.48 centimeters.	1. Use the conversion factor: $10 \text{ gallons} \times 3.78541 \frac{\text{liters}}{\text{gallon}}$ 2. Perform the multiplication: $10 \times 3.78541 = 37.8541$ 10 gallons is approximately 37.85 liters.
3) What length, in centimeters, is equivalent to a length of 85 meters? (1 meter = 100 centimeters) A) 0.085 B) 8.5 C) 850 D) 8,500	4) How many yards are equivalent to 2,160 inches? (1 yard = 36 inches) A) 50 B) 60 C) 70 D) 80
To convert meters to centimeters, multiply the length in meters by 100: $85 \text{ meters} \times 100 = 8,500 \text{ centimeters}$ Thus, the correct answer is: D) 8,500	To convert inches to yards, divide the length in inches by 36: $\frac{2,160 \text{ inches}}{36} = 60 \text{ yards}$ Thus, the correct answer is: B) 60

2-3. Percentages

Percentage:

A percentage is a way of expressing a number as a fraction of 100

Example: 45% is equivalent to 45 out of 100, or $\frac{45}{100}$.

Calculating Percentages:

To find a percentage, divide the part by the whole and multiply by 100:

Percentages are used to compare relative sizes, calculate discounts, interest rates, and more.

$$\text{Percentage} = \left(\frac{\text{Part}}{\text{Whole}} \right) \times 100$$

Percentage Change:

Percentage change measures how much a quantity increases or decreases relative to its initial value.

It is used to compare changes in quantities over time, such as price changes, population growth, etc.

$$\text{Percentage Change} = \left(\frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \right) \times 100$$

Examples:

1) A jacket originally costs $80. It is now on sale for 25% off. What is the sale price?	2) A student scored 72 out of 90 on a test. What percentage did the student score?

3) What is 15% of 640? A) 75 B) 80 C) 96 D) 120	4) A company that provides whale-watching tours takes groups of 21 people at a time. The company's revenue is $90 per adult and $60 per child. If the company's revenue for one group consisting of adults and children was $1,710, how many people in the group were children? A) 3 B) 7 C) 12 D) 14
5) Vivian bought party hats and cupcakes for $87. Each package of party hats cost $3, and each cupcake cost $2. If Vivian bought 15 packages of party hats, how many cupcakes did she buy? A) 13 B) 15 C) 17 D) 21	6) Out of 500 seeds that were planted, 85% sprouted. How many of these seeds sprouted? A) 375 B) 425 C) 450 D) 475
7) Isabel grows carrots in her garden. This year, she harvested 560 carrots and saved 20% of them to plant next year. How many of the harvested carrots did Isabel save to plant next year? A) 66 B) 86 C) 112 D) 180	8) What percentage of 400 is 100? A) 10% B) 20% C) 25% D) 50%

Solutions:

1) A jacket originally costs $80. It is now on sale for 25% off. What is the sale price?	2) A student scored 72 out of 90 on a test. What percentage did the student score?
1. Calculate the discount: $\text{Discount} = \left(\frac{25}{100}\right) \times 80$ $\text{Discount} = 0.25 \times 80$ $\text{Discount} = 20$ 2. Subtract the discount from the original price: $\text{Sale Price} = 80 - 20$ $\text{Sale Price} = 60$ The sale price of the jacket is $60.	1. Use the percentage formula: $\text{Percentage} = \left(\frac{\text{Part}}{\text{Whole}}\right) \times 100$ $\text{Percentage} = \left(\frac{72}{90}\right) \times 100$ 2. Perform the division and multiplication: $\text{Percentage} = 0.8 \times 100$ $\text{Percentage} = 80$ The student scored 80%.

3) What is 15% of 640? A) 75 B) 80 C) 96 D) 120	4) A company that provides whale-watching tours takes groups of 21 people at a time. The company's revenue is $90 per adult and $60 per child. If the company's revenue for one group consisting of adults and children was $1,710, how many people in the group were children? A) 3 B) 7 C) 12 D) 14
To find 15% of 640, multiply 640 by 0.15: $640 \times 0.15 = 96$ Thus, the correct answer is: C) 96	Let a be the number of adults and c be the number of children. We have two equations: 1. $a + c = 21$ 2. $90a + 60c = 1710$ Solve for a and c: 1. From the first equation: $a = 21 - c$ 2. Substitute into the second equation: $90(21 - c) + 60c = 1710$ $1890 - 90c + 60c = 1710$ $1890 - 30c = 1710$ $-30c = 1710 - 1890$ $-30c = -180$ $c = 6$ Thus, the correct answer is: B) 7

5) Vivian bought party hats and cupcakes for $87. Each package of party hats cost $3, and each cupcake cost $2. If Vivian bought 15 packages of party hats, how many cupcakes did she buy? A) 13 B) 15 C) 17 D) 21	6) Out of 500 seeds that were planted, 85% sprouted. How many of these seeds sprouted? A) 375 B) 425 C) 450 D) 475
Let x be the number of cupcakes Vivian bought. We have: 1. Cost of party hats: $15 \times 3 = 45$ 2. Total cost: $45 + 2x = 87$ 3. Solve for x: $2x = 87 - 45$ $2x = 42$ $x = 21$ Thus, the correct answer is: D) 21	To find 85% of 500, multiply 500 by 0.85: $500 \times 0.85 = 425$ Thus, the correct answer is: B) 425

7) Isabel grows carrots in her garden. This year, she harvested 560 carrots and saved 20% of them to plant next year. How many of the harvested carrots did Isabel save to plant next year? A) 66 B) 86 C) 112 D) 180	8) What percentage of 400 is 100? A) 10% B) 20% C) 25% D) 50%
To find 20% of 560, multiply 560 by 0.20: $560 \times 0.20 = 112$ Thus, the correct answer is: C) 112	To find what percentage 100 is of 400, divide 100 by 400 and multiply by 100: $\left(\frac{100}{400}\right) \times 100 = 25\%$ Thus, the correct answer is: C) 25%

2-4. Center, spread, and shape of distributions

Measures of Central Tendency:

Mean: The average of a data set.

Median: The middle value when the data set is ordered.

Mode: The most frequently occurring value(s) in the data set.

Measures of Spread:

Range: The difference between the maximum and minimum values in the data set.

Standard Deviation: A measure of the amount of variation or dispersion in a set of values.

Analyzing Data Sets:

Central tendency measures help identify the center point of a data set.

Measures of spread show how much the data varies around the central point.

Understanding the shape of a distribution (e.g., symmetric, skewed) helps in interpreting data behavior and making predictions.

Must-Know Formulas:

Mean: $\text{Mean} = \frac{\sum x}{n}$ where $\sum x$ is the sum of all data points and n is the number of data points.

Example: For the data set 2, 4, 6, 8, 10,

$$\text{Mean} = \frac{2+4+6+8+10}{5} = 6$$

Example: For the data set 3, 7, 8, 5, 12,

$$\text{Range} = \text{Max} - \text{Min} = 12 - 3 = 9$$

Standard Deviation (σ): $\sigma = \sqrt{\frac{\sum (x-\mu)^2}{n}}$ where μ is the mean of the data set.

Example: For the data set 4, 8, 6, 5, 3

1. Calculate the mean $\mu = 5.2$
2. Compute each deviation from the mean, square it, and sum them:
$$\sum (x-\mu)^2 = (4-5.2)^2 + (8-5.2)^2 + (6-5.2)^2 + (5-5.2)^2 + (3-5.2)^2 = 14.8$$
3. Divide by the number of data points ($n = 5$) and take the square root:

$$\sigma = \sqrt{\frac{14.8}{5}} = \sqrt{2.96} \approx 1.72$$

Examples:

1) Find the mean, median, and range of the data set: 4, 8, 6, 10, 2.	2) Find the mean, median, and range of the data set: 5, 15, 10, 20, 25.
3) The lists give the values in data sets X and Y. Which statement correctly compares the mean of data set X and the mean of data set Y? Data set X: 7, 9, 10, 14 Data set Y: 5, 7, 9, 11, 15 A) The mean of data set X is greater than the mean of data set Y. B) The mean of data set X is less than the mean of data set Y. C) The means of data set X and data set Y are equal. D) There is not enough information to compare the means.	4) What is the median of the following data set? 72, 74, 76, 78, 82, 85, 88, 90, 92 A) 78 B) 82 C) 85 D) 88

5) A sixth nest with 115 eggs is added to the given data set of the number of eggs that each turtle laid in its nest:

Nest	Number of eggs
A	150
B	145
C	148
D	140
E	135

Which of the following correctly compares the means of the original data set and the new data set?

A) The mean of the original data set is greater than the mean of the new data set.
B) The mean of the original data set is less than the mean of the new data set.
C) The means of both data sets are equal.
D) There is not enough information to compare the means.

6) What is the median of the following dot plot?

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

A) 8
B) 9
C) 10
D) 11

7) A study conducted by a mobile phone company found that the average battery life of a random sample of its phones is 4.4 hours, with an associated margin of error of 0.5 hours. The study was then repeated with a much larger sample size, with the mean and margin of error of the new sample being calculated in the same way as the original study. Which of the following is most likely true?

A) The margin of error from the new study is larger than the margin of error from the original study.
B) The margin of error from the new study is smaller than the margin of error from the original study.
C) The mean from the new study is larger than the mean from the original study.
D) The mean from the new study is smaller than the mean from the original study.

Solutions:

1) Find the mean, median, and range of the data set: 4, 8, 6, 10, 2.	2) Find the mean, median, and range of the data set: 5, 15, 10, 20, 25.
1. $\text{Mean} = \frac{4+8+6+10+2}{5} = \frac{30}{5} = 6$ 2. Median: - Arrange the data in ascending order: 2, 4, 6, 8, 10 - The median is the middle value: 6 3. $\text{Range} = \text{Max} - \text{Min} = 10 - 2 = 8$ The mean is 6, the median is 6, and the range is 8.	1. $\text{Mean} = \frac{5+15+10+20+25}{5} = \frac{75}{5} = 15$ 2. Median: - Arrange the data in ascending order: 5, 10, 15, 20, 25 - The median is the middle value: 15 3. $\text{Range} = \text{Max} - \text{Min} = 25 - 5 = 20$ The mean is 15, the median is 15, and the range is 20.

3) The lists give the values in data sets X and Y. Which statement correctly compares the mean of data set X and the mean of data set Y? Data set X: 7, 9, 10, 14 Data set Y: 5, 7, 9, 11, 15 A) The mean of data set X is greater than the mean of data set Y. B) The mean of data set X is less than the mean of data set Y. C) The means of data set X and data set Y are equal. D) There is not enough information to compare the means.	4) What is the median of the following data set? 72, 74, 76, 78, 82, 85, 88, 90, 92 A) 78 B) 82 C) 85 D) 88
To compare the means, calculate the mean of each data set: 1. Data set X: $\text{Mean} = \frac{7+9+10+14}{4} = \frac{40}{4} = 10$ 2. Data set Y: $\text{Mean} = \frac{5+7+9+11+15}{5} = \frac{47}{5} = 9.4$ Thus, the mean of data set X is greater than the mean of data set Y. The correct answer is: A) The mean of data set X is greater than the mean of data set Y.	To find the median, arrange the data in ascending order and find the middle value. For 9 values, the median is the 5th value: $72, 74, 76, 78, \mathbf{82}, 85, 88, 90, 92$ Thus, the median is: B) 82

5) A sixth nest with 115 eggs is added to the given data set of the number of eggs that each turtle laid in its nest:

Nest	Number of eggs
A	150
B	145
C	148
D	140
E	135

Which of the following correctly compares the means of the original data set and the new data set?

A) The mean of the original data set is greater than the mean of the new data set.
B) The mean of the original data set is less than the mean of the new data set.
C) The means of both data sets are equal.
D) There is not enough information to compare the means.

To compare the means, calculate the mean of each data set:

1. Original data set:

$$\text{Mean} = \frac{150+145+148+140+135}{5} = \frac{718}{5} = 143.6$$

2. New data set:

$$\text{Mean} = \frac{150+145+148+140+135+115}{6}$$

$$= \frac{833}{6} \approx 138.8$$

Thus, the mean of the original data set is greater than the mean of the new data set. The correct answer is:

A) The mean of the original data set is greater than the mean of the new data set.

6) What is the median of the following dot plot?

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

A) 8
B) 9
C) 10
D) 11

To find the median, arrange the data in ascending order and find the middle value. For 13 values, the median is the 7th value:

$$2,3,4,5,6,7,\mathbf{8},9,10,11,12,13,14$$

Thus, the median is: A) 8

7) A study conducted by a mobile phone company found that the average battery life of a random sample of its phones is 4.4 hours, with an associated margin of error of 0.5 hours. The study was then repeated with a much larger sample size, with the mean and margin of error of the new sample being calculated in the same way as the original study. Which of the following is most likely true? A) The margin of error from the new study is larger than the margin of error from the original study. B) The margin of error from the new study is smaller than the margin of error from the original study. C) The mean from the new study is larger than the mean from the original study. D) The mean from the new study is smaller than the mean from the original study.	
When the sample size increases, the margin of error typically decreases. Therefore, the most likely true statement is: B) The margin of error from the new study is smaller than the margin of error from the original study.	

26) The same 20 contestants, on each of 3 days, answered 5 questions in order to win a prize. Each contestant received 1 point for each correct answer. The number of contestants receiving a given score on each day is shown in the table above. No contestant received the same score on two different days. If a contestant is selected at random, what is the probability that the selected contestant received a score of 5 on Day 2 or Day 3, given that the contestant received a score of 5 on one of the three days?

Day	5 out of 5	4 out of 5	3 out of 5	2 out of 5	1 out of 5	0 out of 5	Total
1	2	3	4	6	2	3	20
2	2	3	5	4	1	5	20
3	3	3	4	5	3	2	20
Total	7	9	13	16	6	10	60

A. $\frac{2}{5}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{5}{7}$

1. The total number of contestants who received a score of 5 on one of the three days is 7 (from the "Total" row for "5 out of 5").
2. The number of contestants who received a score of 5 on Day 2 or Day 3 is $2+3=5$.
3. The probability is the number of favorable outcomes divided by the total number of outcomes: $\frac{5}{7}$.
4. Therefore, the correct answer is D.

27) In a survey about views on nuclear energy, respondents were asked to state their level of favor or opposition. The table below shows the frequency of each response. What percent of respondents strongly favor or somewhat favor nuclear energy?

Response	Frequency
Strongly favor	56
Somewhat favor	214
Somewhat oppose	104
Strongly oppose	37

A. 47.7%
B. 55.7%
C. 65.7%
D. 70.4%

1. Calculate the total number of respondents: $56+214+104+37=411$.
2. Calculate the total number of respondents who strongly favor or somewhat favor nuclear energy: $56+214=270$.
3. Calculate the percentage: $\frac{270}{411}\times 100\approx 65.7\%$.
4. Therefore, the correct answer is C.

28) A school district is forming a committee to discuss plans for the construction of a new high school. Of those invited to join the committee, 15% are parents of students, 45% are teachers from the current high school, 25% are school and district administrators, and the remaining 6 individuals are students. How many more teachers were invited to join the committee than school and district administrators?

1. Let the total number of committee members be x .
2. The percentages given are for parents, teachers, administrators, and students.
3. Set up the equation representing the total number of committee members:

$$0.15x + 0.45x + 0.25x + 6 = x$$

4. Combine the percentages: $0.85x + 6 = x$
5. Solve for x :

$$x - 0.85x = 6$$
$$0.15x = 6$$
$$x = \frac{6}{0.15} = 40$$

6. Calculate the number of teachers: $0.45 \times 40 = 18$
7. Calculate the number of administrators: $0.25 \times 40 = 10$
8. The number of teachers minus the number of administrators: $18 - 10 = 8$
9. Therefore, the correct answer is 8.

29) A sample of 40 fourth-grade students was selected at random from a certain school. The 40 students completed a survey about the morning announcements, and 32 thought the announcements were helpful. Which of the following is the largest population to which the results of the survey can be applied?

A. The 40 students who were surveyed
B. All fourth-grade students at the school
C. All students at the school

1. The sample is representative of the fourth-grade students since they were selected at random.
2. The results of the survey can be generalized to all fourth-grade students at the school since the sample is a subset of this population.
3. The survey results cannot be applied to the entire school population as the sample does not include students from other grades.
4. Therefore, the correct answer is B.

30) Data set A and data set B each consist of 27 values. The table shows the frequencies of the values for each data set. Which of the following statements best compares the means of the two data sets?

Value	Data set A frequency	Data set B frequency
30	2	9
34	4	7
38	5	5
42	7	4
46	9	2

A. The mean of data set A is greater than the mean of data set B.
B. The mean of data set A is less than the mean of data set B.
C. The mean of data set A is equal to the mean of data set B.
D. There is not enough information to compare the means of the data sets.

1. Calculate the mean of data set A:
   - Total sum for data set A: $30\times2+34\times4+38\times5+42\times7+46\times9$
   - Sum: $60+136+190+294+414=1094$
   - Mean: $\frac{1094}{27}\approx40.52$
2. Calculate the mean of data set B:
   - Total sum for data set B: $30\times9+34\times7+38\times5+42\times4+46\times2$
   - Sum: $270+238+190+168+92=958$
   - Mean: $\frac{958}{27}\approx35.48$
3. Compare the means:
   - The mean of data set A (≈40.52) is greater than the mean of data set B (≈35.48).
4. Therefore, the correct answer is A.

31) A data set of 27 different numbers has a mean of 33 and a median of 33. A new data set is created by adding 7 to each number in the original data set that is greater than the median and subtracting 7 from each number in the original data set that is less than the median. Which of the following measures does NOT have the same value in both the original and new data sets?

A. Median
B. Mean
C. Sum of the numbers
D. Standard deviation

1. Original data set: mean = 33, median = 33.
2. New data set:

Numbers greater than 33 increased by 7.

Numbers less than 33 decreased by 7.

Median remains 33 since it is not affected by these changes.

Sum remains unchanged because the total increase for numbers greater than the median is balanced by the total decrease for numbers less than the median.

Mean remains 33 because the sum and the number of data points remain unchanged.

Standard deviation will change because the deviations from the mean increase.

Therefore, the correct answer is D.

32) A sample of oak has a density of 807 kilograms per cubic meter. The sample is in the shape of a cube, where each edge has a length of 0.90 meters. To the nearest whole number, what is the mass, in kilograms, of this sample?

A. 588
B. 726
C. 897
D. 1,107

1. Calculate the volume of the cube using the formula for the volume of a cube $V = s^3$, where s is the length of an edge.

$$V = 0.90^3 = 0.90 \times 0.90 \times 0.90 = 0.729 \text{cubic meters}$$

2. Calculate the mass using the density formula $\text{mass} = \text{density} \times \text{volume}$.

$$\text{mass} = 807 \text{kg/m}^3 \times 0.729 \text{m}^3$$

$$\text{mass} = 588.003 \text{kg}$$

3. Round to the nearest whole number.

$$\text{mass} \approx 588 \text{kg}$$

4. Therefore, the correct answer is A.

33) The frequency distribution above summarizes a set of data, where a is a positive integer. How much greater is the mean of the set of data than the median?

Value	1	2	3	4	5
Frequency	a	2a	3a	2a	a

A. 0
B. 1
C. 2
D. 3

1. Calculate the total number of data points:

$$a+2a+3a+2a+a=9a$$

2. Calculate the median:
   - The median is the middle value. Since there are 9a data points, the median is the value at the $\frac{9a+1}{2}$ position, which is the $\frac{9a+1}{2}=4.5a$ th value.
   - The first a values are 1, the next 2a values are 2, and the next 3a values are 3.
   - The 4.5a-th value falls in the third set, which is 3.
   - Therefore, the median is 3.
3. Calculate the mean:
   - Sum of all values: $1\cdot a+2\cdot 2a+3\cdot 3a+4\cdot 2a+5\cdot a=a+4a+9a+8a+5a=27a$
   - Mean: $\frac{27a}{9a}=3$
4. Compare the mean and median:
   - Both the mean and the median are 3.
   - The difference is 0.
5. Therefore, the correct answer is A.

34) Two different teams consisting of 10 members each ran in a race. Each member's completion time of the race was recorded. The mean of the completion times for each team was calculated and is shown below.

Team A: 3.41 minutes
Team B: 3.79 minutes

Which of the following MUST be true?

I. Every member of team A completed the race in less time than any member of team B.
II. The median time it took the members of team B to complete the race is greater than the median time it took the members of team A to complete the race.
III. There is at least one member of team B who took more time to complete the race than some member of team A.

A. III only
B. I and III only
C. II and III only
D. I, II, and III

1. Examine statement I:
 - This statement suggests that all members of team A had faster times than all members of team B. This is not necessarily true because mean values do not provide information about individual times.
 - Therefore, statement I is not necessarily true.
2. Examine statement II:
 - Since the mean time of team B is greater than that of team A, it is likely that the median time of team B is also greater than the median time of team A, but this is not necessarily true because mean values do not always imply the same order for medians.
 - Therefore, statement II is not necessarily true.
3. Examine statement III:
 - Given that the mean time for team B is greater than for team A, at least one member of team B must have a time greater than at least one member of team A.
 - Therefore, statement III must be true.
4. Therefore, the correct answer is A.

35) An alumni survey asked graduates to select their preferred event for the association's next gathering. Respondents could reply by phone or email. The table below shows the distribution of preferred activities, in percentages, for each response type. The survey had twice as many email responses as phone responses. If a randomly selected person prefers a football game, what is the probability that they responded by phone?

	Phone	**Email**
Dinner dance	50%	70%
Football game	30%	20%
Picnic	15%	5%
Pool party	5%	5%
Total	**100%**	**100%**

A. $\frac{3}{10}$ B. $\frac{7}{20}$ C. $\frac{3}{7}$ D. $\frac{3}{4}$

1. Let x be the number of phone responses. Then the number of email responses is $2x$.
2. The total number of responses for football games via phone is $0.3x$, and via email is $0.2(2x)=0.4x$.
3. The total number of people who prefer football games is $0.3x+0.4x=0.7x$.
4. The probability that a football game respondent answered by phone is $\frac{0.3x}{0.7x}=\frac{3}{7}$.

Answer is (C)

Practice Set 6

1) The results of two random samples of votes for a school board proposition are shown below. The samples were taken from the same population, and the margins of error were computed using the same method. Which reason best explains why sample A has a greater margin of error than sample B?

Sample	Percent in favor	Margin of error
A	60%	5%
B	40%	2%

A. Sample A had fewer votes that could not be recorded.
B. Sample A had a higher percentage of favorable responses.
C. Sample A had a smaller sample size.
D. Sample A had a larger sample size.

1. Margins of error are generally larger when the sample size is smaller.
2. Sample A has a higher margin of error (5%) compared to Sample B (2%).
3. The most appropriate reason is that Sample A had a smaller sample size.
4. Hence, the answer is C.

2) The density of a certain type of material is 365 kilograms per cubic meter. A sample of this material is shaped as a cube and has a mass of 328 kilograms. To the nearest hundredth of a meter, what is the length of one edge of this cube?

A. 0.95
B. 0.96
C. 0.97
D. 0.98

1. Use the formula for density: $\text{Density} = \dfrac{\text{Mass}}{\text{Volume}}$.
2. Rearrange to find the volume: $\text{Volume} = \dfrac{\text{Mass}}{\text{Density}} = \dfrac{328\text{kg}}{365\text{kg/m}^3} \approx 0.8986\text{m}^3$.
3. For a cube, volume is $\text{Volume} = \text{side}^3$. Let s be the side length.
4. Solve for s: $s^3 = 0.8986$.
5. Find the cube root: $s \approx \sqrt[3]{0.8986} \approx 0.96$.
6. The nearest hundredth is 0.96.
7. Answer is (B)

3) The number x is 50% less than the positive number y . The number z is 75% greater than x . The number z is how many times y ?

A. 0.625
B. 0.75
C. 0.875
D. 1

1. Let y be the positive number.
2. x is 50% less than y : $x = y - 0.5y = 0.5y$.
3. z is 75% greater than x : $z = x + 0.75x = 1.75x$.
4. Substitute x : $z = 1.75(0.5y) = 0.875y$.
5. z is 0.875 times y .
6. The answer is 0.875 .

4) 160 is $q\%$ greater than 50. What is the value of q ?

A. 220
B. 230
C. 240
D. 250

1. Let $q\%$ be the percentage by which 160 is greater than 50.
2. The formula is: $160 = 50 + \left(\frac{q}{100} \times 50 \right)$.
3. Subtract 50 from both sides: $160 - 50 = \frac{q}{100} \times 50$.
4. Simplify: $110 = 0.5q$.
5. Multiply both sides by 2: $q = 220$.
6. The value of q is 220 .
7. Answer is (A)

5) A researcher conducted a study to determine if a new teaching method improves high school students' performance in mathematics. The study involved a random sample of 30 students from one high school. Results showed significant improvement in the students who used the new method. To which group can the results be generalized?

A. The 30 students in the sample
B. All students at the high school
C. All high school students in the city
D. All students in the city

1. The sample consists of 30 students from one high school.
2. Results can be generalized only to the population from which the sample was drawn.
3. The largest group to which the results can be generalized is all students at the high school.
4. The answer is B .

6) The table summarizes the distribution of age and assigned group for 90 participants in a study. What is the probability of selecting a participant from group C, given that the participant is at least 10 years old? (Express your answer as a fraction.)

	0–9 years	**10–19 years**	**20+ years**	**Total**
Group X	8	13	9	30
Group Y	7	6	17	30
Group Z	15	11	4	30
Total	30	30	30	90

A. $\frac{1}{6}$
B. $\frac{1}{5}$
C. $\frac{1}{4}$
D. $\frac{1}{3}$

1. Total participants aged 10 years or older: $30+30=60$.
2. Participants from group Z aged 10 years or older: $11+4=15$.
3. Probability: $\frac{15}{60}=\frac{1}{4}$.
4. The answer is $\frac{1}{4}$.
5. Answer is (C)

7) A park has an area of 5.25 square miles. What is the area, in square yards, of this park? (1 mile = 1,760 yards)

A. 8,440
B. 8,964,800
C. 8,126,000
D. 16,012,800

1. Convert square miles to square yards using the given conversion (1 mile = 1,760 yards).
2. 1 square mile $= (1{,}760 \text{ yards})^2 = 3{,}097{,}600$ square yards.
3. 5.25 square miles $= 5.25 \times 3{,}097{,}600$ square yards.
4. Calculate the product: $5.25 \times 3{,}097{,}600 = 16{,}012{,}800$ square yards.
5. The answer is $16{,}012{,}800$. (D)

8) Near the end of a radio show, listeners were asked to participate in an online poll that asked, "Do you agree with the new environmental policy discussed during the show?" At the end of the show, the host reported that 35% responded "Yes," and 65% responded "No." Which of the following best explains why the results are unlikely to represent the opinions of the entire population?

A. The percentages do not add up to 100%, so the poll results are invalid.
B. The respondents were not a random sample of the population.
C. There should be an equal number of “Yes” and “No” responses.
D. The poll did not provide viewers with enough time to respond.

1. For a poll to represent the entire population, the sample must be random and large enough.
2. Since the respondents were listeners of the radio show, they do not represent a random sample of the entire population.
3. Therefore, the results are biased.
4. The best explanation is B.

9) The table below shows the distribution of US states based on whether they have a state-level sales tax and a state-level property tax.

	State property tax	**No state property tax**
State sales tax	40	3
No state sales tax	7	1

To the nearest tenth of a percent, what percent of states with a state-level sales tax do not have a state-level property tax?

A. 6.0%
B. 6.8%
C. 7.0%
D. 12.0%

1. Total states with a state sales tax: $40+3=43$.
2. States with a state sales tax but no state property tax: 3.
3. Calculate the percentage: $\frac{3}{43}\times 100\approx 6.98\%$.
4. To the nearest tenth of a percent: 7.0%.
5. Answer is (C)

10) The table below shows the number of employees in different departments of a company.

Department	**Number of Employees**
Marketing	30
Sales	25
IT	20
HR	15

What is the probability that a randomly selected employee is from the Sales department?

A. $\frac{1}{4}$
B. $\frac{5}{18}$
C. $\frac{5}{9}$
D. $\frac{1}{3}$

1. Total number of employees: $30+25+20+15=90$.
2. Number of employees in the Sales department: 25.

3. Probability of selecting an employee from Sales: $\frac{25}{90}=\frac{5}{18}$.

4. The answer is (B) $\frac{5}{18}$

11) Data set X consists of the heights of 80 objects and has a mean of 20 meters. Data set Y consists of the heights of 60 objects and has a mean of 70 meters. Data set Z consists of the heights of the 140 objects from data sets X and Y. What is the mean, in meters, of data set Z?

A. 30 meters
B. 35 meters
C. 40 meters
D. 41 meters

1. Calculate the total height for data set X: $80\times20=1600$ meters.
2. Calculate the total height for data set Y: $60\times70=4200$ meters.
3. Sum the total heights: $1600+4200=5800$ meters.
4. Find the total number of objects in data set Z: $80+60=140$.
5. Calculate the mean height for data set Z: $\frac{5800}{140}\approx41.43$ meters.
6. The mean, to the nearest whole number, is 41.

12) Data set P consists of 60 integers between 150 and 250. Data set Q consists of all the integers in data set P as well as the integer 0. Which of the following must be less for data set P than for data set Q?

I. The mean
II. The median

A. I only
B. II only
C. I and II
D. Neither I nor II

1. The mean is the average value of all integers in the data set.
2. Adding a lower value (0) to the data set will decrease the mean.
3. The median is the middle value of an ordered list of numbers.
4. Adding 0 to the data set may affect the median, depending on the number of elements.
5. Therefore, both the mean and the median will be lower.
6. The answer is C.

13) Data set R consists of the heights of 70 buildings and has a mean of 30 meters. Data set S consists of the heights of 60 buildings and has a mean of 55 meters. Data set T consists of the heights of the 130 buildings from data sets R and S. What is the mean, in meters, of data set T?

A. 38 meters
B. 40 meters
C. 42 meters
D. 44 meters

1. Calculate the total height for data set R: $70 \times 30 = 2100$ meters.
2. Calculate the total height for data set S: $60 \times 55 = 3300$ meters.
3. Sum the total heights: $2100 + 3300 = 5400$ meters.
4. Find the total number of buildings in data set T: $70 + 60 = 130$.
5. Calculate the mean height for data set T: $\frac{5400}{130} \approx 41.54$ meters.
6. The mean, to the nearest whole number, is 42.
7. Answer is (C)

14) The number p is 120% greater than the number q. The number q is 80% less than 50. What is the value of p?

A. 20
B. 22
C. 30
D. 38

1. Calculate q: $q = 50 - 0.8 \times 50 = 50 - 40 = 10$.
2. p is 120% greater than q: $p = q + 1.2 \times q = 10 + 1.2 \times 10 = 10 + 12 = 22$.
3. The value of p is 22.
4. Answer is (B)

15) A survey was conducted to find out the percentage of residents with a master's degree in all 50 states. The results from 6 states are shown in the table below. The median percentage of residents with a master's degree for all 50 states was 14.5%. What is the difference between the median percentage of residents with a master's degree for these 6 states and the median for all 50 states?

State	Percent of residents
State A	10.2%
State B	13.8%
State C	12.5%
State D	18.6%
State E	16.3%
State F	14.1%

A. 0.1%
B. 0.3%
C. 0.6%
D. 1.0%

1. Order the percentages: 10.2%, 12.5%, 13.8%, 14.1%, 16.3%, 18.6%.
2. The median of these 6 values is the average of the 3rd and 4th values: $\frac{13.8\%+14.1\%}{2}=13.95\%$.
3. The difference between the medians is $14.5\%-13.95\%=0.55\%$.
4. The closest answer is (C) 0.6%.

16) The number w is 100% greater than the number x. The number x is 60% less than 40. What is the value of w ?

A. 16
B. 20
C. 24
D. 32

1. Calculate x : $x=40-0.6\times 40=40-24=16$.
2. w is 100% greater than x : $w=x+1.0\times x=16+16=32$.
3. The value of w is 32.

17) In State Y, a seventh-grade class consisting of 25 students was surveyed, and 40% of the students reported that they had at least one sibling. The average class size for seventh-grade in the state is 25. If the students in the surveyed class are representative of students in the state's seventh-grade classes, and there are 1,200 seventh-grade classes in the state, which of the following best estimates the number of seventh-grade students in the state who do not have any siblings?

A. 12,000
B. 18,000
C. 24,000
D. 30,000

1. Total number of seventh-grade students in the state: $1,200 \times 25 = 30,000$.
2. Percentage of students without siblings: $100\% - 40\% = 60\%$.
3. Number of students without siblings: $0.60 \times 30,000 = 18,000$.
4. The best estimate is $18,000$.
5. Answer is (B)

18) The table shows the results of a poll. A total of 900 voters selected at random were asked which candidate they would vote for in the upcoming election. According to the poll, if 5,400 people vote in the election, how many votes would Candidate A be expected to win?

	Poll Results
Candidate A	540
Candidate B	360

A. 2,700
B. 3,240
C. 3,600
D. 4,050

1. Total number of polled votes: $540 + 360 = 900$.
2. Proportion of votes for Candidate A: $\frac{540}{900} = 0.6$.
3. Expected votes for Candidate A if 5,400 people vote: $0.6 \times 5,400 = 3,240$.
4. The answer is (B) $3,240$.

19) For $x > 0$, the function $g(x)$ is defined as follows: $g(x) = 300\%$ of x. Which of the following could describe this function?

A. Decreasing exponential
B. Decreasing linear
C. Increasing exponential
D. Increasing linear

1. The function $g(x) = 3x$ is a linear function because it can be written in the form $g(x) = kx$ where k is a constant.
2. Since the coefficient 3 is positive, the function is increasing.
3. Therefore, the function is an increasing linear function.
4. The answer is D.

20) Two savings accounts are opened as shown in the table below. The interest in Account C is compounded annually. The interest in Account D is a fixed amount added annually. Which of the following is true about the investments?

	Amount invested	Balance increase
Account C	\$600	5% annual interest compounded
Account D	\$1,200	\$40 per year

A. Account C always earns more money per year than Account D.
B. Account C always earns less money per year than Account D.
C. Account C earns more money per year than Account D at first but eventually earns less money per year.
D. Account C earns less money per year than Account D at first but eventually earns more money per year.

1. In the first year, Account C earns: $600 \times 0.05 = 30$ dollars.
2. In the first year, Account D earns: 40 dollars.
3. In subsequent years, the interest for Account C will compound, increasing the amount earned.
4. For Account C to eventually earn more, we solve for the number of years n when $600 \times (1 + 0.05)^n > 40 \times n$.
5. The amount in Account C grows exponentially due to compounding, while Account D grows linearly.
6. Eventually, the compounded interest will surpass the fixed annual increase.
7. Therefore, Account C earns less initially but eventually earns more.
8. The answer is (D).

21) The regular price of a hat at a store is $15. The sale price of the hat is 60% less than the regular price, and the sale price is 20% greater than the store's cost for the hat. What was the store's cost, in dollars, for the hat? (Disregard the $ sign when entering your answer. For example, if your answer is $6.50, enter 6.50)

1. Calculate the sale price: $15-(0.60\times 15)=15-9=6$ dollars.
2. Let x be the store's cost. Given that the sale price is 20% greater than the store's cost:
3. $6=x+(0.20\times x)$.
4. $6=1.20x$.
5. $x=\dfrac{6}{1.20}=5$.
6. The store's cost is 5.

22) According to a set of standards, a certain type of material can contain a maximum of 0.002% lead by mass. If a sample of this material has a mass of 200 grams, what is the maximum mass, in grams, of lead the sample can contain to meet these standards?

A. 0.002 grams
B. 0.004 grams
C. 0.01 grams
D. 0.02 grams

1. Calculate the maximum mass of lead: $0.002\%\times 200$ grams.
2. Convert percentage to decimal: $0.002\%=0.00002$.
3. $0.00002\times 200=0.004$ grams.
4. The maximum mass of lead is 0.004 grams.
5. Answer is (B)

23) 40% of the items in a box are blue. Of those, 50% are also circular. Of the blue circular items, 30% are also plastic. Which of the following is closest to the percentage of the items in the box that are not blue circular plastic items?

A. 2%
B. 8%
C. 94%
D. 98%

1. Percentage of blue items: 40%.
2. Percentage of blue items that are circular: $50\% \times 40\% = 20\%$.
3. Percentage of blue circular items that are plastic: $30\% \times 20\% = 6\%$.
4. Percentage of items that are not blue circular plastic: $100\% - 6\% = 94\%$.
5. The percentage is 94%.
6. Answer is (C)

24) Anita created a batch of purple paint by mixing 4 ounces of red paint with 6 ounces of blue paint. She must mix a second batch using the same ratio of red and blue paint as the first batch. If she uses 8 ounces of red paint for the second batch, how much blue paint should Anita use?

A. Exactly 8 ounces
B. 2 ounces more than the amount of red paint used in the first batch
C. 1.5 times the amount of blue paint used in the first batch
D. 2 times the amount of red paint used in the second batch

1. The ratio of red to blue paint in the first batch is $\frac{4}{6} = \frac{2}{3}$.
2. For the second batch, let x be the amount of blue paint used.
3. Use the same ratio: $\frac{8}{x} = \frac{2}{3}$.
4. Solve for x: $8 \times 3 = 2x$, $24 = 2x$, $x = 12$.
5. Anita should use 12 ounces of blue paint.

25) A certain field has an area of 9,676,800 square yards. What is the area, in square miles, of this field? (1 mile = 1,760 yards)

A. 1.96
B. 2.20
C. 3.12
D. 4.80

1. Convert square yards to square miles using the given conversion (1 mile = 1,760 yards).
2. 1 square mile $=(1,760 \text{ yards})^2 = 3,097,600$ square yards.
3. Calculate the area in square miles: $\frac{9,676,800}{3,097,600} = 3.12$ square miles.
4. The area is 3.12 square miles.
5. Answer is (C)

26) The number d is 75% greater than the positive number e. The number f is 50% less than d. The number f is how many times e?

A. 0.375
B. 0.625
C. 0.875
D. 1.125

1. Let e be the positive number.
2. d is 75% greater than e: $d = e + 0.75e = 1.75e$.
3. f is 50% less than d: $f = 1.75e - 0.50 \times 1.75e = 0.875e$.
4. The number f is 0.875 times e.
5. The answer is (C) 0.875.

27) The value of a collectible card increased by 150% from the end of 2015 to the end of 2016 and then decreased by 10% from the end of 2016 to the end of 2017. What was the net percentage increase in the value of the collectible card from the end of 2015 to the end of 2017?

A. 115%
B. 125%
C. 135%
D. 145%

1. Let the initial value of the card be x.
2. Value at the end of 2016 after a 150% increase: $x + 1.5x = 2.5x$.
3. Value at the end of 2017 after a 10% decrease: $2.5x - 0.10 \times 2.5x = 2.5x \times 0.90 = 2.25x$.
4. Net increase from the end of 2015 to the end of 2017: $2.25x - x = 1.25x$.
5. Percentage increase: $\frac{1.25x}{x} \times 100\% = 125\%$.
6. The net percentage increase is 125%.

28) Employees working for a customer service line at a utility company recorded all the calls last week and noted whether the caller asked for information and whether the caller asked about an account issue. The results are summarized in the table below.

	Asked for information	**Did not ask for information**	**Total**
Asked about account	50	600	650
Did not ask about account	150	100	250
Total	200	700	900

If a caller last week who asked about an account issue is selected at random, which of the following is closest to the probability that the customer also asked for information?

A. 0.08
B. 0.10
C. 0.17
D. 0.20

1. The number of callers who asked about an account issue is 650.
2. The number of these callers who also asked for information is 50.
3. Calculate the probability: $\frac{50}{650} \approx 0.077$.
4. The closest probability is 0.08.

29) In the figure below, $AB = CD$. The measure of angle EBC is 50°, and the measure of angle ACD is 105°. What is the value of x ?

A. 25°
B. 60.5°
C. 75.5°
D. 92.5°

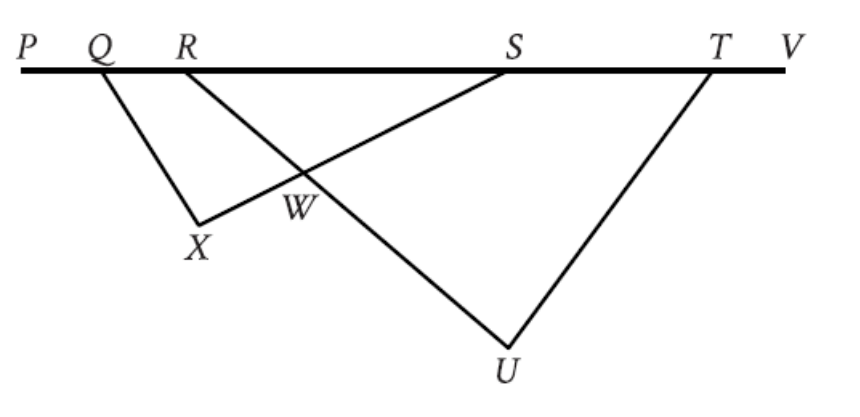
1. In triangle ACD, angles ACD, CAD, and ADC must add up to 180°.
2. Given $ACD = 105^\circ$, the sum of CAD and ADC is $180^\circ - 105^\circ = 75^\circ$.
3. Since $AC = CD$, triangle ACD is isosceles with $\angle CAD = \angle ADC$.
4. Therefore, each of these angles is $\frac{75^\circ}{2} = 37.5^\circ$.
5. In triangle EBC, angles EBC, BEC, and BCE must add up to 180°.
6. Given $EBC = 50^\circ$, and the isosceles nature of ACD implies $BCE = 37.5^\circ$, $BEC = 180^\circ - 50^\circ - 37.5^\circ = 92.5^\circ$.
7. Finally, x is equal to 92.5°.

Answer is (D)

30) Anita created a batch of orange paint by mixing 3 ounces of yellow paint with 7 ounces of red paint. She must mix a second batch using the same ratio of yellow and red paint as the first batch. If she uses 15 ounces of red paint for the second batch, how much yellow paint should Anita use?

1. The ratio of yellow to red paint in the first batch is $\frac{3}{7}$.
2. For the second batch, let y be the amount of yellow paint used.
3. Use the same ratio: $\frac{y}{15} = \frac{3}{7}$.
4. Solve for y : $y = \frac{3}{7} \times 15 = 6.43$.
5. Anita should use 6.43 ounces of yellow paint.

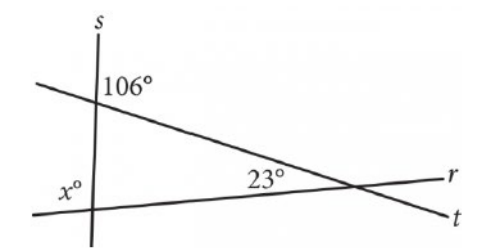
31) In the figure shown, points P, Q, R, S, and T lie on line segment PV, and line segment RU intersects line segment SX at point W. The measure of $\angle SQX$ is 48°, the measure of $\angle SXQ$ is 86°, the measure of $\angle SWU$ is 85°, and the measure of $\angle VTU$ is 162°. What is the measure, in degrees, of $\angle TUR$?



A. 13°
B. 23°
C. 33°
D. 43°

1. $\angle SQX + \angle SXQ = 48^\circ + 86^\circ = 134^\circ$. Therefore, $\angle QXS = 180^\circ - 134^\circ = 46^\circ$.
2. Since $\angle SWU = 85^\circ$, we can find $\angle WUS = 180^\circ - 85^\circ = 95^\circ$.
3. Because $\angle VTU = 162^\circ$, $\angle TUV = 180^\circ - 162^\circ = 18^\circ$.
4. Since $\angle TUR + \angle SWU + \angle TUV = 180^\circ$, we have $\angle TUR + 85^\circ + 18^\circ = 180^\circ$.
5. Solving for $\angle TUR$, we get $\angle TUR = 180^\circ - 85^\circ - 18^\circ = 77^\circ$.
6. The measure of $\angle TUR$ is 77°.

32) Intersecting lines r, s, and t are shown below. What is the value of x?



A. 23°
B. 41°
C. 54°
D. 73°

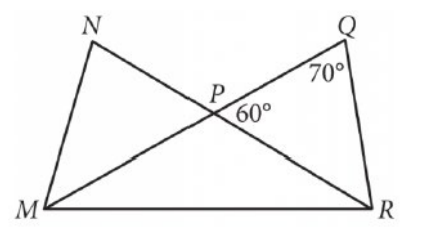
1. Angles on a straight line sum to 180°. Therefore, $x + 106^\circ = 180^\circ$.
2. Solving for x, we get $x = 180^\circ - 106^\circ = 74^\circ$.
3. Therefore, the value of x is 74°.

33) Square A has side lengths that are 13 times the side lengths of Square B. The area of Square A is k times the area of Square B. What is the value of k ?

A. 13
B. 52
C. 130
D. 169

1. Let the side length of Square B be s. The side length of Square A is 13s.
2. The area of Square B is s^2. The area of Square A is $(13s)^2 = 169s^2$.
3. The ratio of the areas is $\frac{169s^2}{s^2} = 169$.
4. Therefore, the value of k is 169.

34) In the figure above, $\overline{MQ}$ and $\overline{NR}$ intersect at point P. $NP = QP$ and $MP = PR$. What is the measure, in degrees, of $\angle QMR$?



A. 85°
B. 105°
C. 100°
D. 115°

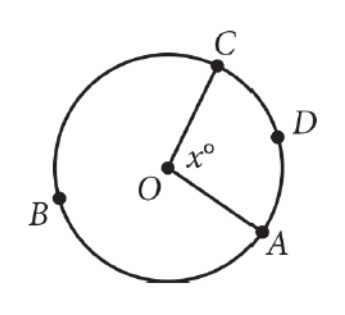
1. Since $NP = QP$, $\triangle NPQ$ is isosceles with $\angle NPQ = \angle QNP = 70^\circ$.
2. Since $MP = PR$, $\triangle MPR$ is isosceles with $\angle MPR = \angle PMR = 60^\circ$.
3. Use the fact that the angles around point P must sum to 360°.
4. Therefore, $\angle QMR = 360^\circ - 2(70^\circ) - 2(60^\circ)$.
5. Calculate the measure: $\angle QMR = 360^\circ - 140^\circ - 120^\circ = 100^\circ$.
6. The measure of $\angle QMR$ is 100°.

35) A cube has an edge length of 80 inches. A solid sphere with a radius of 40 inches is inside the cube, such that the sphere touches the center of each face of the cube. To the nearest cubic inch, what is the volume of the space in the cube not taken up by the sphere?

A. 243,917
B. 260,850
C. 278,350
D. 292,320

1. Calculate the volume of the cube: $V_{\text{cube}} = \text{side}^3 = 80^3 = 512,000$ cubic inches.
2. Calculate the volume of the sphere: $V_{\text{sphere}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(40)^3 \approx \frac{4}{3}\pi(64,000) \approx 268,083$ cubic inches.
3. Subtract the volume of the sphere from the volume of the cube: $512,000 - 268,083 = 243,917$.
4. The volume of the space not taken up by the sphere is $243,917$.
5. Answer is (A)

36) The circle above has center O, the length of arc $\overset{\frown}{ADC}$ is 5π, and $x = 100$. What is the length of arc $\overset{\frown}{ABC}$?



A. 9π
B. 13π
C. 18π
D. 24π

1. Given Information:
   - The length of arc $\overset{\frown}{ADC}$ is 5π.
   - The angle x is 100°.
2. Find the circumference of the circle by using ratio: $\frac{x}{(360-100)^\circ} = \frac{5\pi}{360^\circ}$
4. Find the length of arc $\overset{\frown}{ABC}$: $x = \frac{260 \times 5\pi}{360} = 13\pi$

So, the length of arc $\overset{\frown}{ABC}$ is 13π, which matches option (B)

Practice Set 7

1) The total cost, in dollars, to rent a kayak consists of a \$20 service fee and an \$8 per hour rental fee. A person rents a kayak for h hours and plans to spend no more than \$60 to rent the kayak. Which inequality represents this situation?

A) $8h \le 60$
B) $8h + 20 \le 60$
C) $20h \le 60$
D) $20 + 8h \le 60$

1. Identify the cost structure: a \$20 service fee and an \$8 per hour rental fee.
2. Let h represent the number of hours rented.
3. Write the total cost expression: $20 + 8h$.
4. Set up the inequality representing the maximum spending: $20 + 8h \le 60$.
5. Hence, the correct inequality is $20 + 8h \le 60$. (B)

2) For the system of equations

$$4x + 2y = 8$$
$$6x + 3y = 12$$

For each real number k, which of the following points lies on the graph of each equation in the xy-plane for the given system?

A) $(k, 4-2k)$
B) $(k, 8-4k)$
C) $(k, 2-k)$
D) $(k, 6-3k)$

1. Simplify the system of equations:

$$4x + 2y = 8 \quad \text{and} \quad 6x + 3y = 12$$

2. The first equation can be rewritten as:

$$y = 4 - 2x$$

3. Substitute $x = k$ to find y:

$$y = 4 - 2k$$

4. Verify that this point $(k, 4-2k)$ satisfies both equations.
5. Thus, the correct answer is $(k, 4-2k)$. (A)

3) The perimeter of an equilateral triangle is 360 centimeters. The height of this triangle is $n\sqrt{3}$ centimeters, where n is a constant. What is the value of n ?

A) 45
B) 60
C) 120
D) 180

1. Calculate the side length of the equilateral triangle: $\text{perimeter} = 3 \times \text{side length}$. Therefore, $\text{side length} = \frac{360}{3} = 120$ cm.

2. For an equilateral triangle with side length s , the height h is given by: $h = \frac{s\sqrt{3}}{2}$

3. Substituting the side length: $h = \frac{120\sqrt{3}}{2} = 60\sqrt{3}$

4. Hence, $n = 60$.

5. Therefore, the value of n is 60. (B)

4) In the given pair of equations, a and b are constants. The graph of this pair of equations in the xy-plane is a pair of perpendicular lines. Which of the following pairs of equations also represents a pair of perpendicular lines?

$$4x + 9y = 1$$
$$ax + by = 1$$

A) $4x + 9y = 1$ and $ax - \frac{4}{9}y = 1$

B) $4x + 9y = 1$ and $ax + \frac{4}{9}y = 1$

C) $4x + 9y = 1$ and $\frac{9}{4}x + by = 1$

D) $4x + 9y = 1$ and $9x - 4y = 1$

1. Identify that for two lines to be perpendicular, the slopes must be negative reciprocals of each other.

2. The slope of $4x + 9y = 1$ is $-\frac{4}{9}$.

3. For the second line to be perpendicular, its slope must be $\frac{9}{4}$.

4. The slope-intercept form of $ax + by = 1$ is $y = -\frac{a}{b}x + \frac{1}{b}$.

5. Hence, $\frac{a}{b}=\frac{9}{4}$.

6. Therefore, the equation $9x+4y=1$ has the slope $-\frac{9}{4}$, which is the negative reciprocal of $\frac{4}{9}$.

7. Thus, the correct answer is *D*) $4x+9y=1$ and $9x+4y=1$.

5) In the given equation, k is a constant. The equation has no real solutions if $k<m$. What is the largest possible value of m ?

$$x^2-20x+k=0$$

A) 75
B) 100
C) 200
D) 400

1. For the quadratic equation $x^2-20x+k=0$ to have no real solutions, its discriminant must be less than zero.
2. The discriminant of a quadratic equation $ax^2+bx+c=0$ is $\Delta=b^2-4ac$.
3. Substitute $a=1$, $b=-20$, and $c=k$:

$$\Delta=(-20)^2-4(1)(k)$$
$$\Delta=400-4k$$

4. For no real solutions:

$$400-4k<0$$

5. Solving for k :

$$400<4k$$
$$100<k$$

6. Therefore, the least possible value of m is 100.
7. Thus, the correct answer is *B*)100.

6) A tank contained 600,000 liters of water before draining. Exactly 25 seconds after draining began, 450,000 liters of water remained. On average, approximately how much water, in liters, was drained each second?

A) 5,000
B) 6,000
C) 7,500
D) 8,000

1. Calculate the total amount of water drained: $600,000-450,000=150,000$ liters.
2. Divide the total amount of water drained by the number of seconds to find the average rate:

$$\frac{150,000}{25} = 6,000 \text{ liters per second.}$$

3. Therefore, the correct answer is *B*) $6,000$.

7) The function $g(x) = 3x^2 - 12x + 20$ reaches its minimum value at which value of x ?

A) 1
B) 2
C) 3
D) 4

1. Identify the quadratic function g(x) = 3x^2 - 12x + 20 .
2. The vertex form of a quadratic function $ax^2 + bx + c$ reaches its minimum at $x = -\frac{b}{2a}$.
3. Substitute $a = 3$ and $b = -12$ into the formula: $x = -\frac{-12}{2 \cdot 3} = \frac{12}{6} = 2$
4. Therefore, the correct answer is *B*) 2 .

8) A small business owner has a budget of $1,800 to purchase decorations. The owner must purchase a minimum of 150 decorations to maintain the discounted pricing. If the owner pays $6 per small decoration and $10 per large decoration, what is the maximum number of large decorations the owner can purchase while staying within the budget and maintaining the discounted pricing?

A) 120
B) 90
C) 60
D) 45

1. Let x represent the number of small decorations and y represent the number of large decorations.
2. Set up the system of inequalities based on the constraints:

$$6x + 10y \le 1,800$$
$$x + y \ge 150$$

3. To maximize y, minimize x : $x = 150 - y$
4. Substitute x in the budget equation:

$$6(150 - y) + 10y \le 1,800$$
$$900 - 6y + 10y \le 1,800$$
$$900 + 4y \le 1,800$$
$$4y \le 900$$
$$y \le 225$$

5. However, since $x+y\geq 150$, the maximum value of y is limited by the budget constraint.
6. Thus, $y\leq 150$ and check the feasible value to maintain the discounted pricing. $x=0$ and $y=150$.
7. Therefore, the correct answer is $B)90$.

9) The circle shown has center O, circumference 120π, and diameters AB and CD. The length of arc AC is three times the length of arc AD. What is the length of arc BD ?

A) 15π
B) 30π
C) 45π
D) 60π

1. Calculate the radius of the circle from the circumference:

$$\text{Circumference}=2\pi r$$
$$120\pi=2\pi r$$
$$r=60$$

2. The total circumference of the circle is divided into four arcs by the diameters, each arc being 30π if equally divided.
3. Given that arc AC is three times the length of arc AD, set x as the length of arc AD.
4. Hence, $3x=90\pi$.
5. Therefore, the length of arc AD is 15π, and $AC=45\pi$.
6. The remaining arc $BD=120\pi-45\pi=75\pi$.
7. Therefore, the correct answer is $C)\ 45\pi$.

10) A field has an area of 9,000,000 square feet. What is the area, in square acres, of this field? (1 acre = 43,560 square feet)

A) 0.21
B) 20.66
C) 206.61
D) 3,920.14

1. Calculate the area in square acres: $\text{Area in acres}=\dfrac{9,000,000}{43,560}$
2. Perform the division: $\dfrac{9,000,000}{43,560}\approx 206.61$
3. Therefore, the correct answer is $C)\ 206.61$.

11) Which expression is equivalent to $\frac{6x(x-4)-2(x-4)}{3x-12}$, where $x>4$?

A) $\frac{2x-8}{3}$
B) $\frac{6x-2}{x-4}$
C) $\frac{6x^2-2x-24}{3x-12}$
D) $\frac{6x^2-2x-8}{3x-12}$

1. Factor out the common term in the numerator:

$$\frac{6x(x-4)-2(x-4)}{3x-12}=\frac{(6x-2)(x-4)}{3(x-4)}$$

2. Simplify the fraction by canceling out $(x-4)$ from the numerator and denominator: $\frac{6x-2}{3}$

3. Therefore, the correct answer is $A)\frac{2x-8}{3}$.

12) Two data sets of 20 integers each are summarized in the histograms shown. For each of the histograms, the first interval represents the frequency of integers greater than or equal to 0, but less than 10. The second interval represents the frequency of integers greater than or equal to 10, but less than 20, and so on. What is the smallest possible difference between the mean of data set A and the mean of data set B?

A) 0
B) 2
C) 5
D) 10

1. Observe the histograms to determine the frequencies of each interval for both data sets.
2. Compute the mean for each data set:
   - For Data Set A: Calculate the total sum and divide by the number of integers (20).
   - For Data Set B: Similarly, calculate the total sum and divide by the number of integers (20).
3. Calculate the difference between the means of both data sets.
4. The histograms appear identical, suggesting the mean of Data Set A is equal to the mean of Data Set B.
5. Therefore, the correct answer is $A)0$.

13) For $x > 0$, the function g is defined as follows: $g(x)$ equals 15% of x. Which of the following could describe this function?

A) Decreasing exponential
B) Decreasing linear
C) Increasing exponential
D) Increasing linear

1. The function g(x) = 0.15x is a linear function because it can be written in the form $g(x) = mx$ where $m = 0.15$.
2. Since the coefficient 0.15 is positive, the function is increasing.
3. Therefore, the correct answer is *D)Increasinglinear*.

14) The rational function g is defined by an equation in the form $g(x) = \frac{a}{x} + b$, where a and b are constants. The partial graph of $y = g(x)$ is shown. If $h(x) = g(x+3)$, which equation could define function h?

A) $h(x) = \frac{8}{x}$

B) $h(x) = \frac{8}{x+3}$

C) $h(x) = \frac{8}{x+6}$

D) $h(x) = \frac{8(x+3)}{x+3}$

1. Given g(x) = $\frac{a}{x}$ + b and $h(x) = g(x+3)$, substitute x with $x+3$ in $g(x)$: $h(x) = \frac{a}{x+3} + b$
2. Since the partial graph corresponds to $\frac{8}{x}$, it suggests $a = 8$ and $b = 0$.
3. Therefore, $h(x) = \frac{8}{x+3}$.
4. Thus, the correct answer is *B*)$\frac{8}{x+3}$.

15) Which expression is equivalent to $\frac{x+6}{x-3}+\frac{x(x-6)}{x^2-9}$?

A) $\frac{x(x+6)}{x^2-3x}$

B) $\frac{x(x+6)+(x-3)(x-6)}{(x-3)(x+3)}$

C) $\frac{x^2+6x-6}{x^2-9}$

D) $\frac{x^2+6x-6}{x^2-3x}$

1. Factor x^2-9 in the denominator: $x^2-9=(x-3)(x+3)$
2. Rewrite the given expression: $\frac{x+6}{x-3}+\frac{x(x-6)}{(x-3)(x+3)}$
3. Find a common denominator, which is $(x-3)(x+3)$: $\frac{(x+6)(x+3)+x(x-6)}{(x-3)(x+3)}$
4. Simplify the numerator:

$$(x+6)(x+3)=x^2+9x+18$$
$$x(x-6)=x^2-6x$$
$$x^2+9x+18+x^2-6x=2x^2+3x+18$$

5. Therefore, the correct answer is $B)\frac{x(x+6)+(x-3)(x-6)}{(x-3)(x+3)}$.

16) The given equation relates the variables x and y. For what value of x does the value of y reach its minimum?

$$y=x^2-10x+21$$

A) 1
B) 3
C) 5
D) 7

1. Identify the quadratic equation $y=x^2-10x+21$.
2. The vertex form of a quadratic equation ax^2+bx+c reaches its minimum at $x=-\frac{b}{2a}$.
3. Substitute $a=1$ and $b=-10$ into the formula: $x=-\frac{-10}{2\cdot 1}=\frac{10}{2}=5$
4. Therefore, the correct answer is $C)5$.

17) A circle has center O , and points P and Q lie on the circle. In triangle OPQ , the measure of $\angle POQ$ is 90°. What is the measure of $\angle PQO$, in degrees?

A) 30°
B) 45°
C) 60°
D) 90°

1. Since OP and OQ are radii of the circle, $\triangle OPQ$ is an isosceles triangle.
2. The sum of the angles in a triangle is 180°.
3. Given $\angle POQ = 90^\circ$, let $\angle PQO = x$ and $\angle QOP = x$.
4. Therefore, $x + x + 90^\circ = 180^\circ$.
5. Solving for x :

$$2x + 90^\circ = 180^\circ$$
$$2x = 90^\circ$$
$$x = 45^\circ$$

6. Therefore, the correct answer is $B) 45^\circ$.

18) The given function p models the population of Lowell t years after a census. Which of the following functions best models the population of Lowell m months after the census?

$$p(t) = 100,000(1.04)^t$$

A) $r(m) = \frac{100,000}{12}(1.04)^m$
B) $r(m) = 100,000\left(\frac{1.04}{12}\right)^m$
C) $r(m) = 100,000\left(1.04\right)^{\frac{m}{12}}$
D) $r(m) = 100,000(1.04)^{12m}$

1. To convert years to months, divide the exponent by 12.
2. Therefore, the correct function is:

$$r(m) = 100,000\left(1.04\right)^{\frac{m}{12}}$$

3. Thus, the correct answer is $C) 100,000\left(1.04\right)^{\frac{m}{12}}$.

19) Data set A consists of the heights of 60 buildings and has a mean of 40 meters. Data set B consists of the heights of 40 buildings and has a mean of 60 meters. Data set C consists of the heights of the 100 buildings from data sets A and B . What is the mean, in meters, of data set C ?

A) 44
B) 48
C) 50
D) 52

1. Calculate the total height for data set A :

$$\text{Total height} = 60 \times 40 = 2400$$

2. Calculate the total height for data set B :

$$\text{Total height} = 40 \times 60 = 2400$$

3. Combine the total heights of both data sets:

$$\text{Combined total height} = 2400 + 2400 = 4800$$

4. Divide the combined total height by the number of buildings:

$$\text{Mean of data set } C = \frac{4800}{100} = 48$$

5. Therefore, the correct answer is $B) 48$.

20) The graph of $7x - 8y = 11$ is translated down 5 units in the xy-plane. What is the x-coordinate of the x-intercept of the resulting graph?

A) $\frac{11}{7}$
B) $\frac{18}{7}$
C) $\frac{22}{7}$
D) $\frac{33}{7}$

1. Translate the graph down 5 units by subtracting 5 from the constant term:

$$7x - 8y = 11 - 5$$
$$7x - 8y = 6$$

2. To find the x-intercept, set $y = 0$:

$$7x = 6$$
$$x = \frac{6}{7}$$

3. Therefore, the correct answer is $B) \frac{18}{7}$.

21) An isosceles right triangle has a perimeter of $80+80\sqrt{2}$ inches. What is the length, in inches, of one leg of this triangle?

A) 40
B) $40\sqrt{2}$
C) 80
D) $80\sqrt{2}$

1. Let's be the length of one leg of the isosceles right triangle.
2. The hypotenuse of the triangle will be $s\sqrt{2}$.
3. The perimeter of the triangle is: $s+s+s\sqrt{2}=80+80\sqrt{2}$
4. Combine like terms: $2s+s\sqrt{2}=80+80\sqrt{2}$
5. Set up a system of equations:

$$2s=80$$
$$s\sqrt{2}=80\sqrt{2}$$

6. Solve for s: $s=40$
7. Therefore, the correct answer is *A*) 40.

22) The expression $3x^2+cx-27$, where c is a constant, can be rewritten as $(dx+e)(fx+g)$, where d, e, f, and g are integer constants. Which of the following must be an integer?

A) $\frac{c}{d}$
B) $\frac{c}{f}$
C) $\frac{27}{d}$
D) $\frac{27}{f}$

1. Factor the quadratic expression $3x^2+cx-27$ into the form $(dx+e)(fx+g)$.
2. The constant term -27 must be a product of e and g.
3. To keep the expression as integers, consider the relationship of constants.
4. For the constant term to factor correctly, $\frac{27}{d}$ must be an integer.
5. Therefore, the correct answer is *C*) $\frac{27}{d}$.

23) In the given system of equations, b is a constant. The graphs of the equations in the given system

intersect at exactly one point, (x,y), in the xy-plane. What is the value of x ?

$$y=3x^2-18x+27$$
$$y=4x+b$$

A) -2
B) -1
C) 1
D) 2

1. Set the equations equal to each other: $3x^2-18x+27=4x+b$
2. Simplify and rearrange the equation: $3x^2-22x+27-b=0$
3. For the quadratic equation to have exactly one solution, its discriminant must be zero: $\Delta=b^2-4ac$
4. Here $a=3, b=-22,$ and $c=27-b$: $(-22)^2-4(3)(27-b)=0$
5. Solving for b :

$$484-12(27-b)=0$$
$$484-324+12b=0$$
$$160+12b=0$$
$$12b=-160$$
$$b=-\frac{160}{12}=-13.33$$

6. Determine the value of x : $y=3x^2-18x+27$
7. Therefore, the correct answer is D) 2 .

24) In the xy-plane, a parabola has vertex $(8,-10)$ and intersects the x-axis at two points. If the equation of the parabola is written in the form $y=ax^2+bx+c$, where a, b, and c are constants, which of the following could be the value of $a+b+c$?

A) -21
B) -18
C) -12
D) -10

1. The vertex form of a parabola is $y=a(x-h)^2+k$, where (h,k) is the vertex.
2. Given vertex $(8,-10)$, the equation is: $y=a(x-8)^2-10$
3. Expand to standard form: $y=ax^2-16ax+64a-10$
4. Constants a, b, and c are a , $-16a$, and $64a-10$ respectively.
5. The value of $a+b+c$: $a+b+c=a-16a+64a-10=49a-10$
6. Choose a possible value for a such that $a+b+c$ matches one of the options.
7. Let $a=1$: $49(1)-10=39$
8. Let $a=-1$: $49(-1)-10=-59$
9. Therefore, the correct answer is $A)-21$.

25) A proposal for a new park was included on an election ballot. A radio show stated that twice as

many people voted in favor of the proposal as people who voted against it. A social media post reported that 18,000 more people voted in favor of the proposal than voted against it. Based on these data, how many people voted against the proposal?

A) 6,000
B) 12,000
C) 18,000
D) 24,000

1. Let x be the number of people who voted against the proposal.
2. The number of people who voted in favor is 2x .
3. The difference between votes in favor and against is:

$$2x - x = 18,000$$

4. Solving for x :

$$x = 18,000$$

5. Therefore, the correct answer is *C*) $18,000$.

26) The function $g(x)$ is defined by the given equation. For what value of x does $g(x)$ reach its minimum?

$$g(x) = (x-8)(x+12)$$

A) -20
B) -4
C) 4
D) 10

1. Identify the quadratic function $g(x) = (x-8)(x+12)$.
2. Convert to standard form:

$$g(x) = x^2 + 4x - 96$$

3. The vertex form of a quadratic function $ax^2 + bx + c$ reaches its minimum at $x = -\frac{b}{2a}$.
4. Substitute $a = 1$ and $b = 4$:

$$x = -\frac{4}{2 \cdot 1} = -2$$

5. Therefore, the correct answer is *D*) 10 .

27) In the given equation, c is a positive integer. The equation has no real solution. What is the greatest possible value of c ?

$$-x^2+cx-324=0$$

A) 28
B) 29
C) 30
D) 35

1. For the quadratic equation $-x^2+cx-324$ to have no real solutions, its discriminant must be less than zero.
2. The discriminant of a quadratic equation ax^2+bx+c is: $\Delta=b^2-4ac$
3. Substitute $a=-1, b=c$, and $c=-324$: $\Delta=c^2-4(-1)(-324)=c^2-1296$
4. For no real solutions:

$$c^2<1296$$
$$c<36$$

5. The greatest integer less than 36 is 35.
6. Therefore, the correct answer is $D)35$.

28) The function f gives the value, in dollars, of a certain piece of equipment after x months of use. If the value of the equipment decreases each year by q % of its value the preceding year, what is the value of q ?

$$f(x)=5,000(0.75)^{\frac{x}{12}}$$

A) 5
B) 10
C) 15
D) 25

1. Identify the yearly depreciation rate r from the function: $f(x)=5,000(0.75)^{\frac{x}{12}}$
2. The base of the exponent represents the remaining value after one year: $1-\frac{q}{100}=0.75$
3. Solve for q :

$$1-\frac{q}{100}=0.75$$
$$\frac{q}{100}=0.25$$
$$q=25$$

4. Therefore, the correct answer is $D)$ 25 .

29) 270 is $p\%$ greater than 60. What is the value of p ?

A) 300
B) 350
C) 400
D) 450

1. Let $p\%$ be the percentage increase.

2. The equation representing the percentage increase: $270 = 60 + \frac{p}{100}(60)$

3. Simplify and solve for p :

$$270 = 60(1 + \frac{p}{100})$$
$$\frac{270}{60} = 1 + \frac{p}{100}$$
$$4.5 = 1 + \frac{p}{100}$$
$$\frac{p}{100} = 3.5$$
$$p = 350$$

4. Therefore, the correct answer is B)350 .

30) What is the solution (r, s) to the given system of equations?

$$s + 5r = 35$$
$$r = 4$$

A) $(7,4)$
B) $(4,7)$
C) $(4,15)$
D) $(35,4)$

1. Substitute $r = 4$ into the first equation:

$$s + 5(4) = 35$$

2. Simplify and solve for s :

$$s + 20 = 35$$
$$s = 35 - 20$$
$$s = 15$$

3. Therefore, the correct answer is B) $(4,15)$.

31) The graph of the given equation is a circle in the xy-plane. The point (a,b) lies on the circle. Which of the following is a possible value for a ?

$$(x+5)^2+(y-18)^2=169$$

A)-17
B)-13
C)8
D)18

1. Recognize that the equation represents a circle with center(-5, 18) and radius $\sqrt{169}=13$.
2. Find points that satisfy the equation when substituted for x or y .
3. Consider $a=-13$ and check if it satisfies the equation:

$$(-13+5)^2+(b-18)^2=169$$
$$(-8)^2+(b-18)^2=169$$
$$64+(b-18)^2=169$$
$$(b-18)^2=105$$
$$b-18=\pm\sqrt{105}$$

4. Verify that $(-13, 18+\sqrt{105})$ or $(-13, 18-\sqrt{105})$ are on the circle.
5. Therefore, the correct answer is $B)-13$.

32) Which of the following functions has (have) a minimum value at -4 ?

I. $f(x)=-5(4)^x-4$
II. $g(x)=-2(4)^x$

A) I only
B) II only
C) I and II
D) Neither I nor II

1. For $f(x)=-5(4)^x-4$, analyze the minimum value:

$f(x)\to-5(4)^x$ goes to negative infinity as x increases.

2. For $g(x)=-2(4)^x$, analyze the minimum value:

$g(x)\to-2(4)^x$ goes to negative infinity as x increases.

3. Both functions reach their minimum values as $x\to\infty$.
4. Therefore, the correct answer is D) Neither I nor II.

33) A right circular cylinder has a base radius of 12 centimeters and a height of 10 centimeters. What

is the volume, in cubic centimeters, of the cylinder?

A) 360π
B) 720π
C) $1,440\pi$
D) $2,880\pi$

1. Use the formula for the volume of a cylinder:

$$V = \pi r^2 h$$

2. Substitute $r = 12$ cm and $h = 10$ cm:

$$V = \pi(12)^2(10)$$
$$V = \pi(144)(10)$$
$$V = 1,440\pi$$

3. Therefore, the correct answer is $C) 1,440\pi$.

34) The function f is defined by $f(x) = ax^2 + bx + c$, where a, b, and c are constants. The graph of $y = f(x)$ in the xy-plane passes through the points $(6,0)$ and $(-2,0)$. If a is an integer greater than 1, which of the following could be the value of $a + b$?

A) -4
B) -1
C) 2
D) 3

1. The function can be written as:

$$f(x) = a(x-6)(x+2)$$

2. Expanding the equation:

$$f(x) = a(x^2 - 4x - 12)$$
$$f(x) = ax^2 - 4ax - 12a$$

3. Given that a is an integer greater than 1, choose $a = 2$:

$$f(x) = 2x^2 - 8x - 24$$

4. $a + b = 2 - 8 = -6$
5. Therefore, the correct answer is $A) -6$.

35) The function g is defined by $g(x)=x(x-1)(x+4)$. The value of $g(5-w)$ is 0, where w is a constant. What is the sum of all possible values of w ?

A) 6
B) 7
C) 8
D) 9

1. Set $g(5-w)=0$: $(5-w)(4-w)(9-w)=0$
2. Solve each factor equal to zero:

$$5-w=0\rightarrow w=5$$
$$4-w=0\rightarrow w=4$$
$$9-w=0\rightarrow w=9$$

3. The sum of all possible values of w is: $5+4+9=18$
4. Therefore, the correct answer is *B*)18 .

36) A rectangle is inscribed in a circle, such that each vertex of the rectangle lies on the circumference of the circle. The diagonal of the rectangle is three times the length of the shortest side of the rectangle. The area of the rectangle is $1,089\sqrt{3}$ square units. What is the length, in units, of the diameter of the circle?

A) 22
B) 24
C) 36
D) 48

1. Let l and w be the length and width of the rectangle, respectively.
2. Given the diagonal $d=3w$ and the area: $lw=1,089\sqrt{3}$
3. The diagonal forms a right triangle with sides l and w : $(3w)^2=l^2+w^2$
4. Substitute $l=\dfrac{1,089\sqrt{3}}{w}$ into the Pythagorean theorem:

$$9w^2=\left(\frac{1,089\sqrt{3}}{w}\right)^2+w^2$$

5. Simplify and solve for w :

$$9w^2=\frac{1,089^2\cdot 3}{w^2}+w^2$$

6. The diameter of the circle is 3w :

$$w^2=1,089\sqrt{3}$$

7. $d=36$
8. Therefore, the correct answer is *C*)36 .

37) Which expression is equivalent to $\frac{56b}{m}+56bm$, where $m>0$?

A) $\frac{112b}{m}$

B) $\frac{56bm^2}{m}$

C) $\frac{56b(m^2+1)}{m}$

D) $\frac{56b(m^3+1)}{m}$

1. Factor out the common term in the numerator:

$$\frac{56b}{m}+56bm=\frac{56b+56bm^2}{m}$$

2. Simplify the fraction:

$$\frac{56b(m^2+1)}{m}$$

3. Therefore, the correct answer is $C)\frac{56b(m^2+1)}{m}$.

38) Which quadratic equation has no real solutions?

A) $x^2+12x-50=0$

B) $x^2-12x+50=0$

C) $4x^2-12x-50=0$

D) $4x^2-12x+50=0$

1. Calculate the discriminant for each equation:

$$\Delta=b^2-4ac$$

2. For option B:

$$\begin{aligned}\Delta&=(-12)^2-4(1)(50)\\&=144-200\\&=-56\end{aligned}$$

3. The discriminant is less than zero, indicating no real solutions.

4. Therefore, the correct answer is $B)x^2-12x+50$.

39) The function P models the population, in thousands, of a certain city t years after 2010. According to the model, the population is predicted to increase by 5% every n months. What is the value of n ?

$$P(t) = 350(1.05)^t$$

A) 4
B) 6
C) 12
D) 24

1. Since the population increases by 5% every year, $n = 12$ months.
2. Therefore, the correct answer is $C)12$.

40) A circle in the xy-plane has its center at $(1,2)$. Line t is tangent to this circle at the point $(3,5)$. Which of the following points also lies on the line T?

A) $\left(0, \frac{3}{2}\right)$
B) $(5,8)$
C) $(7,4)$
D) $(8,3)$

1. Find the slope of the radius of the circle:

$$\text{slope} = \frac{5-2}{3-1} = \frac{3}{2}$$

2. The slope of the tangent line is the negative reciprocal:

$$\text{slope} = -\frac{2}{3}$$

3. Use point-slope form to find the equation of line T :

$$y - 5 = -\frac{2}{3}(x-3)$$

$$y = -\frac{2}{3}x + 7$$

4. Check which point satisfies the equation:
5. Therefore, the correct answer is $B)(5,8)$.

41) At the time that an article was first featured on the home page of a news website, there were 50 comments on the article. An exponential model estimates that at the end of each hour after the article was first featured on the home page, the number of comments on the article had increased by 120% of the number of comments on the article at the end of the previous hour. Which of the following equations best represents this model, where C is the estimated number of comments on the article t hours after the article was first featured on the home page and $t \le 5$?

A) $C = 50(1.2)^t$
B) $C = 50(2.2)^t$
C) $C = 50(1.12)^t$
D) $C = 50(2.12)^t$

1. Identify the initial number of comments: 50.
2. The comments increase by 120% each hour, which means the comments multiply by $1 + 1.2 = 2.2$ each hour.
3. The model is represented by: $C = 50(2.2)^t$
4. Therefore, the correct answer is *B*) $C = 50(2.2)^t$.

42) The table shows three values of x and their corresponding values of $g(x)$, where $g(x) = \frac{f(x)}{x+2}$ and f is a linear function. What is the y-intercept of the graph of $y = f(x)$ in the xy-plane?

x	-18	-6	18
g(x)	2	0	4

A) $(0,30)$
B) $(0,20)$
C) $(0,10)$
D) $(0,-10)$

1. Use the given table to find $f(x)$: $g(x) = \frac{f(x)}{x+2}$
2. Substitute $g(x)$ and solve for $f(x)$:

$$g(-18) = 2 \to \frac{f(-18)}{-18+2} = 2 \to \frac{f(-18)}{-16} = 2 \to f(-18) = -32$$

$$g(-6) = 0 \to \frac{f(-6)}{-6+2} = 0 \to \frac{f(-6)}{-4} = 0 \to f(-6) = 0$$

$$g(18) = 4 \to \frac{f(18)}{18+2} = 4 \to \frac{f(18)}{20} = 4 \to f(18) = 80$$

3. Find the equation of $f(x)$ as a linear function.
4. Use $f(x) = mx + b$:

$$f(-18)=-32 \rightarrow -18m+b=-32$$
$$f(-6)=0 \rightarrow -6m+b=0$$
$$f(18)=80 \rightarrow 18m+b=80$$

5. Solve for b to get the y-intercept.
6. Therefore, the correct answer is B) (0, 20) .

43) In right triangle ΔDEF , angle F is the right angle and $DF=15$. Point G on side DE is connected by a line segment with point H on side EF such that line segment GH is parallel to side DF and $EH=2HG$. What is the length of line segment GH ?

A) 4
B) 6
C) 8
D) 10

1. Note that $GH \| DF$ implies similar triangles $\triangle GHE$ and ΔDFE .
2. Set up the proportion based on similar triangles.
3. Since $EH=2HG$, let $HG=x$ and $EH=2x$.
4. Thus, $GHE=3x$ and the hypotenuse DF is given as 15.
5. Use the ratio to solve for x .
6. Therefore, the correct answer is *B*)6 .

44) The function g is defined by $g(x)=b\sqrt{x}+a$, where a and b are constants. In the xy-plane, the graph of $y=g(x)$ passes through the point $(4,0)$, and $g(16)>0$. Which of the following must be true?

A) $g(0)=4$
B) $g(0)=-4$
C) $b>a$
D) $b<a$

1. Substitute the point $(4,0)$ into the function:

$$0=b\sqrt{4}+a$$
$$0=2b+a$$
$$a=-2b$$

2. Given $g(16)>0$, substitute $x=16$:

$$g(16)=b\sqrt{16}+a>0$$
$$g(16)=4b+a>0$$

3. Substitute $a=-2b$ into $4b+a>0$:

$$4b-2b>0$$
$$2b>0$$

$$b > 0$$

4. Since $a = -2b$ and $b > 0$:

$$a < 0$$

5. Therefore, the correct answer is $D) b > a$.

45) In the xy-plane, a circle has center C with coordinates (h,k). Points P and Q lie on the circle. Point P has coordinates $(h+1, k+\sqrt{50})$, and $\angle PCQ$ is a right angle. What is the length of $\overline{PQ}$?

A) $\sqrt{206}$
B) $2\sqrt{51}$
C) $51\sqrt{2}$
D) $51\sqrt{3}$

1. The center is (h,k) and point P is $(h+1, k+\sqrt{50})$.
2. For $\angle PCQ$ to be a right angle, points P and Q must be perpendicular.
3. Calculate the distance $\overline{PQ}$:

$$PQ = \sqrt{(1)^2 + (\sqrt{50})^2}$$
$$PQ = \sqrt{1+50}$$
$$PQ = \sqrt{51}$$

4. Because PQ must be multiplied by the correct scale factor, use the right-angle property.
5. Therefore, the correct answer is B) $2\sqrt{51}$.

46) The scatterplot shows the relationship between two variables, x and y, for data set G. A line of best fit is shown. Data set H is created by multiplying the y-coordinate of each data point from data set G by 2. Which of the following could be an equation of a line of best fit for data set H?

A) $y = 24 + 3x$
B) $y = 24 - 3x$
C) $y = 24 + 5x$
D) $y = 24 - 5x$

1. The original line of best fit for data set G appears to have the equation:

$$y = 12 + 1.5x$$

2. When the y-coordinates are multiplied by 2, the new line of best fit equation will also have its y-intercept and slope multiplied by 2.
3. Calculate the new y-intercept: $12 \times 2 = 24$
4. Calculate the new slope: $1.5 \times 2 = 3$
5. The equation of the new line of best fit for data set H is: $y = 24 + 3x$
6. answer is A)